Single camera, multiple-plane PIV by synthetic refocusing

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Abstract An extended particle image velocimetry (PIV) measurement technique is presented. Images of a flow are captured by a single camera. The synthetic aperture method is applied to extract 3D information out of the 2D image, which contains multiple views on the flow imaged by a self-made lens array consisting of twenty-one doublet lenses. Tracer particles in the flow are illuminated by five equally spaced light sheets. All particles are imaged at the same time. Three-dimensional, two component velocity fields are determined by refocusing of the 2D image. The field of view is 30mm in diameter. The covered depth range is 50mm in total, given by the light sheet distance of 12.5mm. The lateral resolution of the vector fields is 1.3mm. The depth resolution is given by the light sheet thickness of 1mm. For validation the sensor is applied to a laterally heated, free convective flow. The flow basin is 100x100x100mm\textsuperscript{3}. Temporally averaged velocity data reveals the 3D flow pattern.

1. Introduction

Standard particle image velocimetry (PIV) (e.g. Willert and Gharib 1991) is one of the most common tools in experimental fluid mechanics. PIV provides two components of the flow velocity in two spatial dimensions fixed by an illuminating light sheet. Comprehensive understanding of complex unsteady flow structures often requires 3D velocity data. The development of PIV thereby arising culminates in manifold sensors. In figure 1 we arrange in order the herein presented PIV measurement technique to existing 3D-PIV techniques. After two decades of PIV development two approaches for 3D measurements emerge: I) PIV by light sheet illumination and II) PIV by volumetric illumination. In fact, illumination required for PIV complies with the measurement method. Meaning, someone planning standard 2D-PIV measurements will not choose volumetric illumination, instead of a single light-sheet. Because correct illumination is fundamental for the measurement method, it is reasonable to take it as classification criterion. Another classification of PIV measurement techniques is the number of detectable spatial dimensions (short: D) and the number of detectable velocity components (short: C), see Hinsch (1995) for instance.

Fig.1 Classification of 3D-PIV techniques by light-sheet and volumetric illumination. The encircled synthetic aperture PIV is introduced in the present work.
One characteristic of light-sheet illumination is concentrating the available light energy in one or more planes within the measurement volume. Locally, this enables much higher light intensity compared to volumetric illumination. Further, signals from different, parallel light sheets can be analyzed simultaneously by applying multiplexing techniques. This allows measuring 2D velocity fields on each light-sheet separately. Multiplexing means light from multiple light sheets is collected by a single optics. Signals from different light sheets are encoded differently, e.g. by wavelength, time, phase or defocus blur. Consequently, separated data for individual light sheets can be recovered at the receiving end.

At least six physical quantities allow recovering data from individual light sheets: polarization (Kähler and Kompenhans 2000), wavelength (Mullin et al. 2005, Pick et al. 2009), time (also called scanning light-sheet techniques, Brücker 1995), phase (light sheet holography, Hinsch 1996, Palero et al. 2005), size of defocus blur (defocussing PIV, Liberon et al. 2004) and the parallax (herein presented synthetic aperture PIV). Techniques using volumetric illumination are: holographic PIV (e.g. Hinsch 1996), tomographic PIV (Elsinga et al. 2006), defocussing PIV (Pereira and Gharib 2002) and synthetic aperture PIV by Belden et al. (2010). Stereoscopic PIV, see Arroyo and Greated (1991), is treated as extension to light-sheet illumination techniques. If light sheets are separated by inverse multiplexing, adding at least one further camera would enable detecting the off-plane velocity component on each light sheet. This would extend 3D2C light-sheet PIV to a 3D3C technique resolving all three components of the flow velocity within the light sheets. For techniques present in the light-sheet and the volumetric illumination group requirements of sensor resolution are different. With increasing depth resolution light sheets can be located closer to each other without crosstalk. For a particular resolution light sheets even may be stacked without crosstalk. In this case the measurement volume is illuminated continuously.

In 2010 Belden introduce the synthetic aperture technique to fluid mechanics by combining it to PIV. Especially for synthetic aperture PIV large depth resolution is obtained in the expense of sensor size. Belden et al. use eight cameras mounted in an aluminium frame. All cameras image the same particle-laden flow. Recorded images are recombined by algorithmic known from computer vision in order to obtain depth information, see e.g. Wilburn et al. (2005). Belden et al. present 3D3C velocity fields of a vortex ring proving the performance of SAPIV.

As the parallax is essential for depth estimation in SAPIV, reducing the sensor size inevitably reduces the depth resolution. Sensors using light-sheet illumination require less depth resolution. Measurement positions in depth are forced by light sheets. Combining the SAPIV by Belden with light-sheet illumination enables downsizing of the sensor to a single-camera setup. Only two components of the velocity vector can be provided by this sensor (3D2C). But recovering the single light sheets is straightforward. No inverse multiplexing units (polarization/wavelength division PIV, holography) or sophisticated trigger electronics (time division resp. light sheet scanning PIV) are required. Using one camera allows applying SAPIV to flows that are accessible optically through small viewports only.

The combination of the synthetic aperture technique and PIV is explicated in section 2. The application of the sensor is presented in section 3. Results and Discussion are presented in section 4 followed by Conclusions in section 5.

2. Method

To our knowledge, the synthetic aperture technique is applied to fluid mechanics by Belden et al. (2010) for the first time. In akin synthetic synthetic-aperture radar (SAR) used for topographic mapping (e.g. Leith 1978) a receiver with small aperture is mounted under an airplane. As the receiver moves during imaging, a sequence of temporally shifted images is recorded. All single images are combined in order to synthetize an image at higher resolution than the single ones, which corresponds to a synthetically enlarged aperture. The major difference between SAR and
synthetic-aperture particle image velocimetry (SAPIV) is that image acquisition is realized by a lens array instead of a single moving lens. The imaging process in SAPIV is illustrated in figure 2. Two particles at axial positions $Z_1$ and $Z_2$ are imaged through specialized optics, whose centerpiece is a planar lens array. The evolving image is calculated by the ray tracing code ZEMAX. The aperture of the lenses in the array is small. The depth of field is large enough for imaging both particles sharply. For illustration the particles are colored black and grey. The color does not comply with light energy emitted by the particle. In the following we explicate how the aperture of the optics is enlarged synthetically allowing focusing on particles either in depth $Z_1$ or in depth $Z_2$.

Each lens images the particle pair at a different angle. For the lens on the optical axis of the optics the magnification for both particles is the same. Both particles are imaged onto on dot. For lenses beside the optical axis the magnification is different for each particle. Here the magnification of the black particle is larger than for the grey one. This results in an image shift on the sensor plane. The shift increases for increasing distance of the array lens to the optical axis. We deduce the position of the imaged dots on the sensor plane is a function of the axial particle position $Z$. The image in figure 2 consists of twenty-one sub-images corresponding to twenty-one lenses in the array. In the following, all sub-images are registered, see figure 3. The image sections are cut and laid over the central sub-image. A shift map is applied to the image sections such that all dots corresponding to the black particle overlie; see mid part of figure 3. Because of constructive summation, a large peak appears – the black particle. The evolved image is cropped and normalized to the maximal grey value of 255. The same is repeated for the grey particle with its corresponding shift map. In other words, all sub-images are combined to a single new one of same size as the central sub-image. Both shift maps correspond to a different depth $Z$ and may be used to recalculate particle Z-positions from images captured by the same optics. The recalculation is called refocusing as well.

A threshold is applied to the summed images in order to eliminate destructively summed grey values corresponding to particles at different depth positions. In general, the required threshold
depends on the seeding density in the flow. The obtained shift maps are composed quite simple. There is one shift vector for one complete sub-image, which is only correct for thin light sheets. Further, distortions are excluded. Due to aberrations of real optics image points may deviate from positions calculated for idealized optics. This fact may be overcome by applying a homography, a central projection mapping between two planes, which are formed by a non-linear grids (e.g. Hartley and Zisserman, 2010). The real optics and its alignment relative to the experimental apparatus are sketched in figure 4. As distortion can be considered a field dependent tilt, large field angles are blocked by an aperture plate located in the intermediate image, see figure 4. In this work, the applied homography uses one point in the sub-images and one point in the central sub-image enabling straight forward post-processing. One point homographies only are valid for lenses imaging at diffraction limit (no significant aberrations). This is why doublets are chosen for the lens array.

![Diagram of experimental setup](image)

**Fig.4** Experimental setup for multiple-plane PIV with synthetic aperture technique. A laser beam is expanded to five light sheets by an optical grating, illuminating a convective flow. The optics is aligned normal to the light sheets. All tracer particles present on the light sheets are imaged onto a CCD sensor plane by a lens array at the same time.

In order to calculate the depth uncertainty of the present SAPIV, we assume two particles from different depths are imaged sharply onto the CCD camera, which is absolutely true for low lens apertures. Figure 5 illustrates ray propagation through the lens array. Front lens, field lens and lens array from figure 4 merge in the depicted model lens array.
Two particles in the measurement volume are imaged at different magnification resulting in different heights $h_a$ and $h_b$. The dots on the sensor plane are distinguishable if they are separated by at least one pixel width, $h_a - h_b \geq 1px$. The lens pitch is denoted by $p$, the light sheet location is $Z_i$, the image distance is $b$. The magnification of a particle located on one of the light sheets is given by $\beta(Z_i) = -b/Z_i$. The equation for the shift of the image points reads:

\[
h_a - h_b = p \left( \frac{-b}{Z_i+\delta z} - \frac{-b}{Z_i} \right) \geq 1px
\]  

(1)

With $h_a - h_b = 1px$ the uncertainty of depth calculates to:

\[
\delta z = -b \left( \frac{1}{\beta(Z_i)p} - \frac{1}{\beta(Z_i)} \right)
\]  

(2)

As the lens array in figure 5 is a substitute for the real optics, the image distance $b$ cannot be measured directly. It is determined experimentally by measuring the scale of magnification on adjacent light sheets.

\[
b = -\left( \frac{Z_i-Z_{i+1}}{\beta(Z_i)^{-1}-\beta(Z_{i+1})^{-1}} \right)
\]  

(3)

With 17.4\(\mu\)m pixel size, 3.13mm lens pitch, a light sheet distance of $Z_i-Z_{i+1} = 12.5\text{mm}$ and the present lenses the uncertainty of depth calculates to $\delta z = 5\text{mm}$. The uncertainty $\delta z$ is indirect proportional to the maximum detectable magnitude of parallax; see stereo-optic PIV (Arroyo and Greated, 1991). For increasing pitch $p$ the uncertainty $\delta z$ decreases, equation (2).

As mentioned before, two particles shifted in depth only are distinguishable if their images are separated by at least one camera pixel. But in fact, the depth of focus of the front lens needs to be considered additionally. Particles outside the depth of focus are imaged blurred because of defocus aberration. Further, light propagation through dielectric interfaces, like the wall of the flow basin, induces spherical aberration resulting in an additional blur. For successful refocusing of on-line particles present on different light sheets, particle images should be separated by more than one pixel. In order to estimate the refocusing performance of the sensor in advance, the quantity $h = \beta \cdot p$ can be determined for all light sheets. As $h = 0$ corresponds to the position of the optical axis, values of $h$ equal the shifts of sub-images relative to the center sub-image. Meaning, the whole shift map can be calculated. Image magnification $\beta$ is calculated by laws of geometric optics. With $f$ the total focal length of the optics the conditional equation for $\beta$ holds:
\[ \beta^2 + \left(2 - \frac{Z+b}{D}\right)\beta + 1 = 0 \] (4)

The difference of \( h \) between two evaluated depth positions is a measure for the quality of image separation. Equation (4) is valid for perfect alignment of the lenses in the array, especially in absence of lens tilt. In practice, measured shifts may differ massively from theoretic values. Measured shifts are presented in figure 6.

Intuitively, one may think synthetic aperture refocusing works the better, the larger the number of views (cameras or lenses). Belden et al. (2010) simulate reconstruction quality as a function of the number of used cameras and confirm this assumption. Compared to Belden, we use twenty-one views instead of eight. This should be advantageous for reconstruction – especially at large seeding densities.

Calibration is carried out in-situ under no-flow condition. All light sheets except of one are blocked and one particle image is recorded. This procedure is repeated until one image for each light sheet is acquired. The acquired images are processed by MATLAB. A cross-correlation algorithm is applied to estimate image shifts. In calculated 2D cross-correlation functions the highest peak and its 5 pixel surrounding are cut. The resulting 11x11px section is fitted by an elliptical Gaussian in order to account for non-spherical particle images, see e.g. Raffel (2007). The peak position is calculated by a Levenberg-Marquardt least-squares minimization algorithm. Image shifts are determined at about 0.1px accuracy. Calibrated image shifts are sketched in figure 6. Shifts are given in pixel as a function of the distance between the wall of the flow basin facing the front lens and the light sheet position, \( D = 100 \text{mm} \). The colored line roughly divides two sets of curves. The curves are assigned to outer, inner and central lenses by the colored bar beside the plot and the lens array draft.

Each curve represents the measured image shift of an individual lens relative to the central lens in the array. Shifts are determined at positions of the five light sheets and connected by straight lines. Curves with high level shift correspond to outer lenses in the array, while curves with low level shifts belong to inner lenses; see the colored legend in figure 6. The straight line with zero shifts corresponds to the image of the central lens shifted to itself. It can be considered a test for the correlation algorithm. The maximal shift is 72px at \( Z/D = 0.55 \), corresponding to a lens with maximum distance to the center lens. The smallest shift is 9px at \( Z/D = 0.05 \), corresponding to a lens very close to the center lens. The sensor performance is given by the gradient of the shift function. In figure 6, the smallest difference between adjacent light sheet positions is 1.02px, measured between \( Z/D=0.05 \) and \( Z/D=0.175 \). Because of decreasing magnification \( \beta \) along \( Z \), the largest difference of 8.3px is determined between \( Z/D=0.425 \) and \( Z/D=0.55 \). For refocusing inner lenses of the lens array contribute stronger than outer lenses. This is because of the planar structure of the lens array. Relative to the center lens outer lenses only detect the rim of the joint FOV.
Meaning, they do not image all particles within the FOV. Their contribution to the summation is less. The smallest shift difference in the calibration map should exceed 1px for high reconstruction quality.

3. Experiment

The single camera 3D-PIV technique is applied to a three-dimensional convective flow. The fluid is heated laterally, which may be a relevant configuration for the manufacture of bulk semi-conductor crystals, see for instance Lappa (2005).

The flow is generated in a basin with \( D = 100 \text{mm} \) edge length. One side wall of the basin is connected to the heating, the opposite wall to the cooling circuit. Both circuits are realized by independently working water thermostats (Julabo HC, Germany), connected to the walls by tubes. The remaining four walls of the flow basin are considered almost adiabatic, meaning there is no significant heat transfer to the environment. The adiabatic walls are made from PMMA and therefore are transparent. The temperature controlled walls are made from aluminum. Thermo couples connected to an USB input controller (Meilhaus Electronics) are countersunk centrally in the heated and cooled flow basin walls. They face the fluid and log the wall temperature during measurements. Further, there is a vertical chain of six thermo couples countersunk in an adiabatic wall in order to obtain the vertical temperature distribution. In thermodynamic equilibrium, for the free convective flow the temperature controlled walls exhibit mean temperatures of 76.88°C and 5.86°C respectively. Within the flow basin the temperatures at the bottom and top of the measurement volume are 33.91°C and 44.06°C respectively.

The flow basin is filled completely with a 0.73:0.27 water-glycerin solution resulting in 1.061g/cm3 fluid density at 30°C (Glycerine Producers' Association, 1963). Glycerin is added to the fluid in order to adapt the fluid's density to the used tracer particles. Tracers are produced from polyamide 12 base polymer. The manufacturer's name is Vestosint 1141 (Evonik, Germany). Referring to manufacturer’s data the density of dry tracers is 1.06 g/cm3. The fraction of 55 % of particles ranges from 100μm to 250μm in diameter. With increasing temperature the fluid’s density decreases. The density difference between bottom and top of the measurement volume is about 5×10^{-3} g/cm^3 (Glycerine Producers' Association, 1963). With appropriate viscosity the stationary rising velocity of the used 100μm particles is estimated to be 10μm/s at the uppermost position of the measurement volume.

Laser, receiving optics and the flow basin are mounted on an aluminum frame. Their orientation in the experimental apparatus is illustrated in figure 4. The optics and the camera must be shaded to prevent stray light from exposing the sensitive CCD chip. The laser is a Minilite with 5ns pulse width manufactured by Continuum, USA, whose power is adjusted by controlling the time delay between firing the flash lamp and opening Q-switch. Maximal 30\% of the available laser energy of 25mJ is used for experiments. The repetition rate of the laser is adjusted to 1Hz by an external trigger. The 2mm laser beam is expanded one-dimensionally by a negative cylindrical lens, focal length \( f=-25\text{mm} \). Then the beam propagates a phase grating (Edmund Optics). In a distance of about 300mm behind the grating five diffraction orders are parallelized by a cylindrical lens, focal length \( f=300\text{mm} \). After parallelization the light sheets are equally spaced by 12.5mm. The grating exhibits nearly the same intensity distribution in the +1^{st},0 and −1^{st} diffraction order. In the +2^{nd} and −2^{nd} order the diffracted intensity is lower. Applying neutral density filters for the inner three orders enables same light intensity for all five light sheets. This is important for the refocusing algorithm to work properly. Heterogenic intensity would deteriorate reconstruction quality.

A side wall is viewpoint for the optics. The optics is aligned normal to the wall of the flow basin and to the light sheets. All lenses and the camera are mounted on a rail allowing on-axis shifting relatively to the flow basin. The first lens is an object lens, \( f=50\text{mm} \), located at a working distance of 95mm to the flow basin. Very different working distances are feasible by appropriate choice of
the object lens. If the used object lens is replaced by a zoom lens, the working distance could be varied without changing the alignment of the receiving optics. The object lens forms a demagnified intermediate image of the flow. At the position of the intermediate image an aperture plate with variable diameter enables fine adjusting the final size of the measurement volume. Field of views (FOV) large compared to the extension of the central lens in the lens array only are observable for demagnified imaging. The FOV can be enlarged by reducing magnification $\beta$ by a factor $n < 1$. Following equation (3), magnitude $b$ increases proportional to $n$ in good approximation. Meaning, the depth uncertainty $\delta z$ would increase for enlarging the FOV. Consequently, the light sheet spacing must be enlarged by the same factor $n$ to ensure correct refocusing. For the present setup 30mm FOV is realized. The light sheet spacing must equal at least 10mm for successful refocusing, as $\delta z = \pm 5\text{mm}$ (section 2). The quotient $\text{FOV}/(2\delta z)$ is a sensor constant. It is $\text{FOV}/(2\delta z) = 3$ for the present optics. With $s$ the light sheet spacing the sensor specific design rule is deduced:

$$\text{FOV}/s \leq 3$$  \hspace{1cm} (5)

For covering the total flow basin a light sheet spacing of 33mm is required for successful refocusing. In this case only three light sheets would fit into the flow basin. Further, extending the depth range would lead to additional blur of particle images due to defocus aberration. The maximal feasible FOV for all light sheets lying within the basin is about 65mm at $s = 22.5\text{mm}$. Hardware constraints limit the present diameter of the FOV to 30mm. A camera with smaller pixel and larger chip size would reduce $\delta z$ as well, which enables smaller $s$ spacing and larger FOV.

Behind the cropped intermediate image a positive field lens, $f=50\text{mm}$, is used as diverging lens. The diverging pencil of light rays exits the field lens and is imaged by the lens array. The distance between field lens and array is 30mm. The lens array consists of 21 doublet lenses with focal length $f=9\text{mm}$ and 3mm in diameter by Edmund Optics. The lenses are aligned in a quadratic grid and mounted between two silicon plates. Holes 2mm in diameter are etched into the silicon plates as apertures for the lenses. Lenses are glued into the silicon frame by polydimethylsiloxane (PDMS). The lens array, field lens and aperture plate are mounted in a LINOS micro bench system. The used camera is a Photron APX RS 1024x1024px with $17.8\times17.8\text{mm}^2$ camera chip size. The camera is triggered by the same electronics determining the repetition rate of the laser. For observing large FOV the magnification $\beta$ should be as small as possible, but it is limited by the chip size. The FOV is imaged multiplied onto the CCD chip by the lens array. In the present case the effective chip area used for the FOV is $204\times204\text{px}$, which is 25 times smaller than the chip size. The effective area is given by the number of lenses in the array. Magnification $\beta$ is significantly different for each light sheet due to the large sheet spacing compared to the focal length of the front lens. Knowledge of the exact value of $\beta$ is important to determine particle positions on individual light sheets. Figure 7 shows magnification values measured by imaging a scale.

![Fig.7 Image magnification $\beta$ as a function of depth $Z$ within the flow basin of $D = 100\text{mm}$ edge length.](image-url)
The realized particle density is 0.03 particles per mm³. For the applied optics this results in 0.005 particles per pixel (ppp). This is three times smaller than Belden’s seeding density. As the parallax is much smaller compared to Belden, we reduce the seeding density as well.

After reaching thermodynamic equilibrium the recording is started at 1Hz repetition rate of laser firing and camera exposure. 2048 images are recorded. In figure 8 a 1024x1024px snapshot of particles on all the five light sheets is displayed. The image is composed of twenty-one sub-images corresponding to the same number of lenses. Particles in figure 8 exhibit different size. This is not only due to the variance in particle size, but also due to defocus. Differences in adjacent sub-images indicate different viewing angles.

![Image](image_url)

**Fig.8** 1024x1024px snapshot of tracer particles in the convective flow. The image is formed by twenty-one single lenses. Tracer particles are present on five light sheets equally spaced by 12.5mm. The diameter of the field of view is about 30mm.

All twenty-one sub-images are interpolated at 0.1px steps to allow sub-pixel shifting. Subsequently all sub-images are shifted two-dimensionally by the calibration map, given in figure 6. Summing and thresholding all images enables identification of particles on individual light sheets. In figure 9 the raw central sub-image containing particles from all light sheets is displayed. Investigations reveal no preprocessing of the raw images is necessary. First for all, this is reasoned by selective illumination via light sheets. The mid part of figure 9 is the refocused image at \(Z/D = 0.425\). Destructive summation of the sub-images leads to washed patterns and blurs particles present on the investigated light sheet. Undesired patterns and blur are blocked by applying threshold. The threshold is twelve standard deviations above the mean intensity of the refocused image. The thresholded image is given in figure 9, right.

![Image](image_url)

**Fig.9** Identification of tracer particles at \(Z/D = 0.425\) by refocusing. Left: Unprocessed
204x204px center sub-image containing particles from all light sheets. Mid: Refocused image. Right: Mid image after applying the threshold.

4. Results and Discussion

The processing chain indicated in figure 9 is repeated for every light sheet. Particles in the raw image are assigned to each of the five light sheets. For each light sheet 2048 recordings captured at 1Hz are analyzed. Due to low seeding density a complete velocity field can only be calculated using several recordings. Particle displacements are evaluated by the open source code fluere by K. P. Lynch on sequenced image pairs. The magnification scale is adapted for each light sheet in accordance to figure 7. A three pass algorithm is applied starting at 64x64px, continuing with 32x32px and ending at 16x16px interrogation window size. Windows are weighted uniformly, 50% overlap leads to a smallest window size of 8x8px, which determines the spatial resolution of the vector field to 1.3mm edge length. The depth resolution is given by the light sheet thickness of 1mm. Velocity vectors are validated in MATLAB using the ratio of correlation peaks within the 2D correlation function. Cross correlation is applied for each light sheet separately. All validated two-component vectors are averaged over time. One 2D mean velocity field at \( Z/D = 0.55 \) is given in figure 10 including the orientation of the measurement volume within the flow basin. The measurement volume is composed of five slices starting at \( D/Z = 0.05 \) and ending at the depicted 2D velocity field at \( Z/D = 0.55 \). The lateral extension of measurement slices increases for larger values of \( D/Z \), as image magnification decreases. Dimensions are related to the edge length \( D = 100\text{mm} \) of the flow basin.

![Fig.10 Orientation of the measurement volume within the flow basin. Exemplarily, one temporarily averaged vector field and its color-coded absolute velocity is shown at \( Z/D = 0.55 \), at 8x8px resolution.](image)

The mean velocity \( \bar{v}_{\text{abs}} \) field of the time series visualized in MATLAB is displayed in figure 11. It is composed of five X-Y slices at five depth positions equally spaced by 12.5mm. The edges of the slices form a cuboid conically widening in Z-direction. The peaks of the displayed cones point the direction of the flow. The size of the cones is a measure for the absolute velocity. Identical to figure 10, the magnitude of the temporally averaged velocity is given as color-coded slice for the lowest and highest \( Z/D \). A section of the convective flow is presented, as the complete flow cannot be demagnified arbitrarily, see equation (5). Covering the complete flow would require shifting the receiving optics laterally. The flow is directed upwards. As expected, the flow velocity increases with height X because of buoyant forces acting on the fluid. The flow seems to exhibit a single-roll pattern with its center of rotation underneath the geometric cell center. The systematic velocity uncertainty for both vector components is \( du = dv = 17\mu\text{m/s} \) due to the large pixel size of
17.4µm. Beside the low seeding density, this reasons why the temporarily averaged velocity field is presented. As the fluid’s temperature increases along $X$, the vertical velocity uncertainty $dv$ is even larger in the upper part of the measurement volume. This is a fundamental problem, when investigating convective flows at high temperature differences. In the present case it leads to erroneous velocity data in the upper part of the measurement volume. The stationary sinking velocity of the used tracers is estimated to be near zero in the bottommost region, increasing to 10µm/s in the uppermost region. Reducing the temperature difference or utilizing smaller tracer particles would be a remedy.

![Figure 11](image1.png)

**Fig.11** Section of temporally averaged, 3D velocity field in a laterally heated convective flow. Two-component flow vectors are depicted as cones. Velocity magnitudes $\bar{v}_{abs}$ are given on $Z$-slices bordering the measurement volume. The fluid is a 0.73:0.27 water-glycerin solution. The hot wall ($\theta = 76.88^\circ C$) is at $Y/D = 0$, the cold wall ($\theta = 5.86^\circ C$) at $Y/D = 1$. Five spaced light sheets are used for flow illumination. The velocity field is determined by means of synthetic aperture refocusing with a single camera.

In figure 12 the maximal absolute velocities $\sigma_{abs}$ on each light sheet are plotted along the $Z$-coordinate. As expected, $\sigma_{abs}$ increases monotonically.

![Figure 12](image2.png)

**Fig.12** Magnitudes of maximal, mean velocities $\bar{v}_{abs}$ in the convective flow as a function of $Z/D$. 
5. Conclusions

Synthetic aperture refocusing is combined with particle image velocimetry (PIV). 2D images of a single camera are refocused in order to determine the 3D position of tracer particles in a flow. A lens array is mounted in front of the camera providing twenty-one views with different parallax on the same field of view (FOV). For validation the sensor is applied to a convective flow. The analyzed section of the 100x100x100mm³ flow field exhibits characteristic flow behavior proving correct functionality of the sensor.

In contrast to literature on synthetic aperture PIV (Belden, 2010) describing bulky setups with multiple cameras, a single camera setup is presented. The single camera view enables application of 3D-PIV in environments not accessible for multiple-camera setups, like reactors with low aperture view ports or wind tunnels. The sensor is much handier, but provides poor depth resolution. Refocusing of particle images is only feasible in combination with light sheet illumination. Five light sheets equally spaced by 12.5mm are refocused successfully with doublet lenses 3mm in diameter at an FOV 30mm in diameter. Larger FOV are feasible at larger light sheet spacing s obeying the specific design rule \( \text{FOV}/s \leq 3 \). At unchanged or even reduced spacing s the FOV can be enlarged by a camera with smaller pixel or larger chip size. A modern consumer camera would allow imaging an FOV of about 100mm.

The realized seeding density of 0.005 particles per pixel (ppp) is low compared to existing 3D-PIV techniques, see e.g. Elsinga (2006). Denser particle images could be analyzed, too. Especially the high number of twenty-one views is advantageous, as simulated by Belden (2010), and should allow refocusing in spite of the small parallax of the lens array. The number of views could be even increased by larger camera chips allowing usage of larger lens arrays. Further preprocessing like median filtering and contrast enhancement for instance will improve reconstruction quality. In the present work no image enhancement algorithms are used at all. Even equalizing of histograms between the time step recordings is omitted. As refocusing works fine, image enhancement seems to be redundant for seeding density in the range of 0.005ppp.

The working distance of the sensor can be varied easily, as only the object lens needs to be exchanged. This is a significant advantage over multiple-camera systems for 3D visualization.

Synthetic aperture refocusing is feasible with a single camera. This may encourage further spread of the technique in the field of fluid mechanics, as 3D flow visualization is an attractive tool for analyzing complex flows.

6. References

- Glycerine Producers' Association (1963) Physical properties glycerine and its solutions