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Räumlich und zeitlich aufgelöste lokale und globale Sensitivitätsanalyse der Konsolidation um eine punktförmige Wärmequelle

Aqeel Afzal Chaudhry[1], Jörg Buchwald[2,1], Thomas Nagel[1,3]

[1] Institut für Geotechnik, Technische Universität Bergakademie Freiberg
[2] Helmholtz-Zentrum für Umweltforschung GmbH – UFZ, Leipzig
[3] TUBAF-UFZ Zentrum für Umweltgeowissenschaften

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OUTLINE

Introduction

What really is sensitivity analysis?

Sensitivity analysis

Sensitivity analysis refers to the study of how different sources of uncertainty in the model input contribute to the uncertainty in the model output.

Saltelli et al. 2004

–> Why sensitivity analysis?

- Subsurface –> complex physical phenomena
- Large spatial and temporal scales
- Uncertainties in model inputs

–> How is sensitivity analysis helpful?

- Process understanding
- Model based decision making
- Design optimization

Introduction

–> Use of coupled THM models?

- Assessment of nuclear waste disposal
  - DECOVALEX –> DEvelopment of COupled models and their VALidation against EXperiments
- Reservoir engineering
- Geotechnical engineering

–> Why analytical solution is beneficial?

- Well Understood
- Necessary primary couplings
- Entire parameter space
- Inexpensive

THM consolidation around a point heat source

Analytical solution –> Booker and Savvidou 1985 – Chaudhry et al. 2019
Thermal part –> heat balance

$$(\rho c_p)^{\text{eff}}\dot{T} + \rho_{\text{w}} c_{p,\text{w}}\, \text{grad}\, T \cdot \mathbf{v} - \text{div}\, (K\, \text{grad}\, T) = q_T \tag{1}$$

where

$$(\rho c_p)^{\text{eff}} = \phi \rho_{\text{w}} c_{p,\text{w}} + (1-\phi)\, \rho_{\text{s}} c_{p,\text{s}} \tag{2}$$

$$K = \phi K_{\text{w}} + (1-\phi)\, K_{\text{s}} \tag{3}$$

$$\mathbf{v} = -\frac{k_{\text{s}}}{\mu_{\text{w}}}\, (\text{grad}\, p - \rho_{\text{w}} \mathbf{g}) \tag{4}$$

Hydraulic part –> mass balance

$$\beta \dot{p} - [\phi a_{\text{w}} + (\alpha_{\text{B}} - \phi)\, a_{\text{s}}]\, \dot{T} + \alpha_{\text{B}}\, \text{div}\, \dot{\mathbf{u}} + \text{div}\, \mathbf{v} = q_{\text{H}} \tag{5}$$

Mechanical part –> linear momentum balance

$$\text{div}\, \boldsymbol{\sigma} + \rho \mathbf{g} = \mathbf{0} \tag{6}$$

where

$$\rho = \phi \rho_{\text{w}} + (1-\phi) \rho_{\text{s}} \tag{7}$$

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}' - \alpha_{\text{B}} p \mathbf{1} \tag{8}$$

$$\boldsymbol{\sigma}' = 2G \boldsymbol{\varepsilon}^{\text{el}} + \lambda\, \text{tr}\, \boldsymbol{\varepsilon}^{\text{el}} \mathbf{1} \tag{9}$$

$$\boldsymbol{\varepsilon}^{\text{el}} = \boldsymbol{\varepsilon} - \frac{a_{\text{s}}}{3} \Delta T \mathbf{1} \tag{10}$$

THM consolidation around a point heat source

Assumptions:

1. Spherical symmetry
2. Constant heat flux
3. Thermal, hydraulic, mechanical isotropy
4. No external fluid source –> $q_{\mathrm{H}} = 0$
5. No account of gravity
6. Incompressible solid and fluid phases –> $\alpha_{\mathrm{B}} = 1$ and $\beta = 0$
7. No advection –> unilateral thermal to HM-coupling –> low-permeable

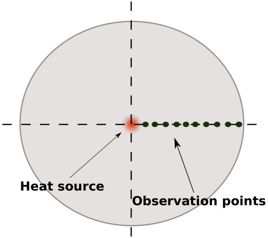


Fig. 1: Setup showing a point heat source embedded in an isotopic fully saturated porous medium and the observation points at which the SA are performed.

Types of coupling

- Primary couplings (addressed in this work)
- Secondary couplings

Heat balance:

$$(\rho c_p)^{\mathsf{eff}}\dot{T} - \operatorname{div}\,(K \operatorname{grad} T) = q_T$$

Mass balance:

$$[\phi a_{\mathsf{w}} + (1-\phi)\,a_{\mathsf{s}}]\,\boxed{\dot{T}} - \operatorname{div}\boxed{\dot{\mathbf{u}}} + \operatorname{div}\left(\frac{k_{\mathsf{s}}}{\mu_{\mathsf{w}}}\operatorname{grad} p\right) = 0$$

Linear momentum balance:

$$\operatorname{div}(\boldsymbol{\sigma}' - \boxed{p}\,\mathbf{1}) = \mathbf{0}$$

$$\boldsymbol{\sigma}' = 2G\boldsymbol{\varepsilon}^{\mathsf{el}} + \lambda \operatorname{tr}\boldsymbol{\varepsilon}^{\mathsf{el}}\mathbf{1}$$

$$\boldsymbol{\varepsilon}^{\mathsf{el}} = \boldsymbol{\varepsilon} - \frac{a_{\mathsf{s}}}{3}\boxed{\Delta T}\,\mathbf{1}$$

THM consolidation around a point heat source

Input data: Callovo-Oxfordian (COx) clay rock – Armand et al. 2017 – Plúa et al. 2020

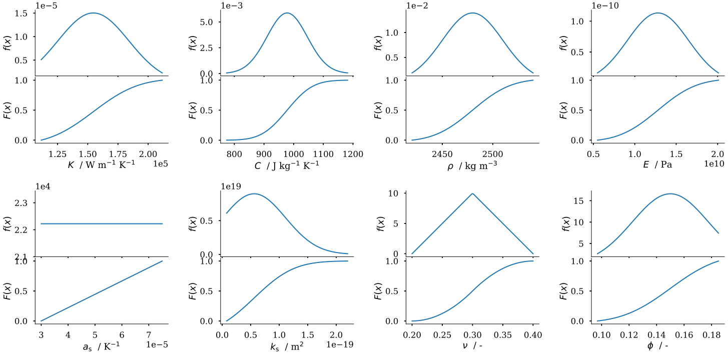


Fig. 2: A priori probability density function (PDF) $f(x)$ and cumulative distribution function (CDF) $F(x)$ for input variables.

Model outputs

Temperature (T) – Pore pressure (p) – Radial displacement (u_r)

$$r \in \{r_1, \ldots, r_{10}\} = \{2\,\text{m}, 4\,\text{m}, 6\,\text{m}, 8\,\text{m}, 10\,\text{m}, 12\,\text{m}, 15\,\text{m}, 20\,\text{m}, 25\,\text{m}, 30\,\text{m}\}$$
$$t \in \{t_1, \ldots, t_{10}\} = \{1\,\text{d}, 7\,\text{d}, 30\,\text{d}, 90\,\text{d}, 180\,\text{d}, 1\,\text{y}, 2\,\text{y}, 3\,\text{y}, 4\,\text{y}, 5\,\text{y}\}$$

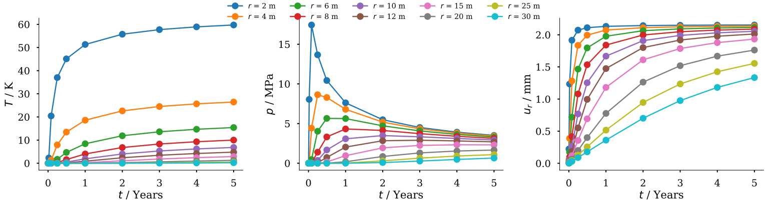
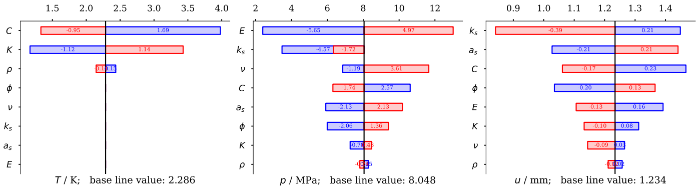
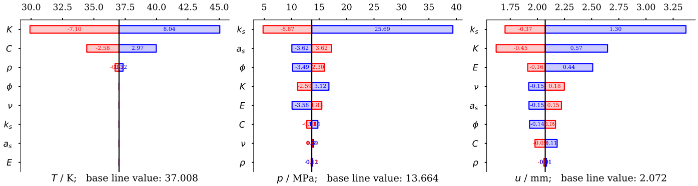


Fig. 3: Analytical solution at $t \in [t_1, t_{10}]$ at $r \in \{r_1, \ldots, r_{10}\}$ for mean values of input parameters

OVAT Screening



(a) $r = 2\,\mathrm{m}$, $t = 7\,\mathrm{d}$



(b) $r = 2\,\mathrm{m}$, $t = 90\,\mathrm{d}$

Fig. 4: Tornado chart for temperature, pressure and displacement close to the heat source at different time points.

OVAT Analysis

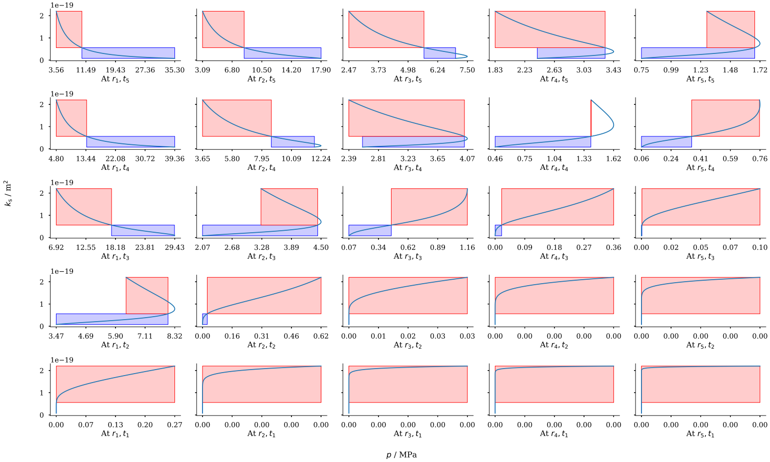
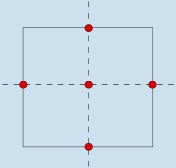


Fig. 5: OVAT analysis for model input k_s and output p

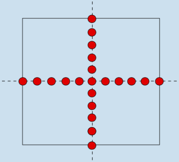
OVAT Analysis

Curse of dimensionality:

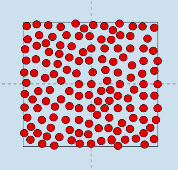
Tornado charts



OVAT analysis



Global sensitivity analysis



Variance-based global sensitivity analysis – Sobol 2001

First order sensitivity index

$$S_i = \frac{V_i(Y)}{V(Y)} \tag{11}$$

Second order sensitivity index

$$S_{ij|i\neq j} = \frac{V_{ij}(Y)}{V(Y)} \tag{12}$$

Total order sensitivity index

$$S_{\mathsf{T}\,i} = S_i + \sum S_{ij} + \sum S_{ij\ldots n} \tag{13}$$

- Python library SALib –> https://github.com/SALib/SALib
- Sampling based on improved Sobol's sequence –> Saltelli et al. 2010
- No. of samples = 100 000, No. of model re-evaluations = 1 800 000

Global Sensitivity Analysis

Spatio-temporal sensitivity maps—temperature

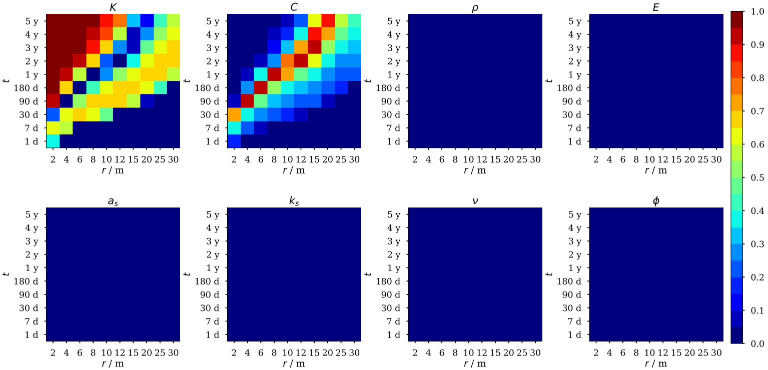


Fig. 6: S_i for T for all combinations of r and t.

Spatio-temporal sensitivity maps—temperature

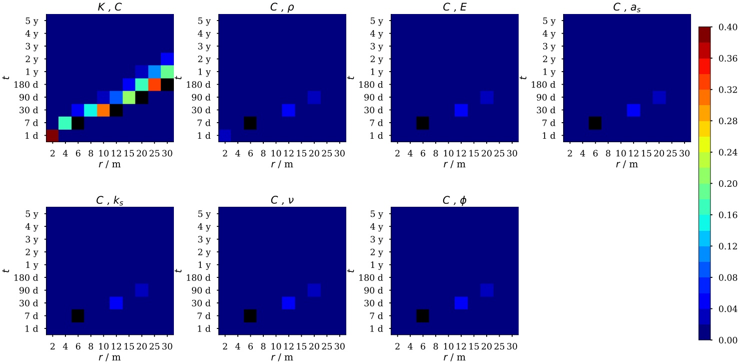


Fig. 7: S_{ij} for T for all combinations of r and t.

Global Sensitivity Analysis

Spatio-temporal sensitivity maps—pressure

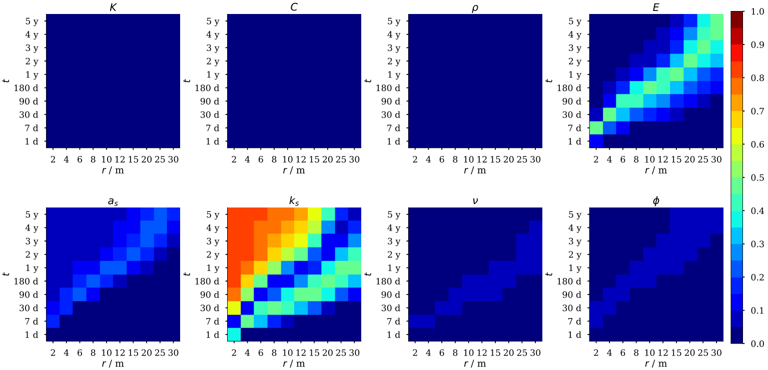


Fig. 8: S_i for p for all combinations of r and t.

Spatio-temporal sensitivity maps—pressure

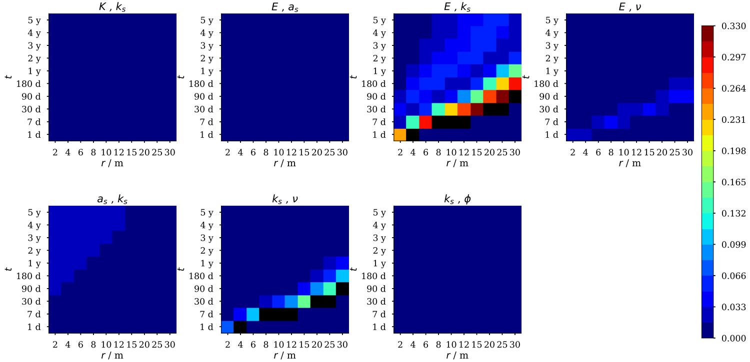


Fig. 9: S_{ij} for p for all combinations of r and t.

Spatio-temporal sensitivity maps—radial displacement

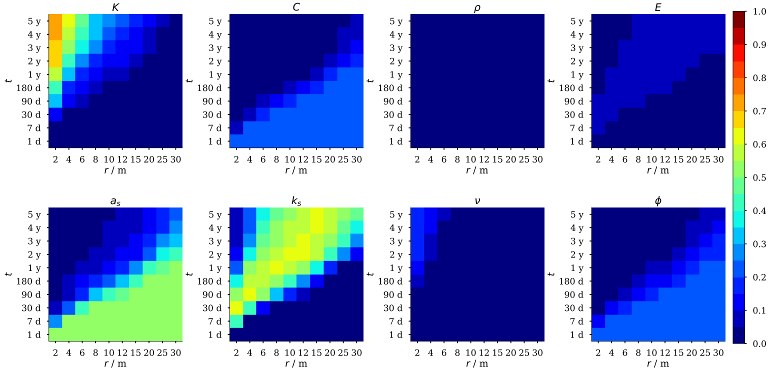


Fig. 10: S_i for u_r for all combinations of r and t.

Global Sensitivity Analysis

What about secondary couplings?

- Example: Intrinsic permeability –> Vol. strain
- Analytical solution not suitable
- Numerical techniques.
  Computational cost?
- Alternative to full-scale GSA?
  Surrogate models *

* J Buchwald, AA Chaudhry, K Yoshioka, O Kolditz, S Attinger, and T Nagel. "DoE-based history matching for probabilistic uncertainty quantification of thermo-hydro-mechanical processes around heat sources in clay rocks". In: *International Journal of Rock Mechanics and Mining Sciences* 134 (2020), p. 104481

Conclusions

- OVAT screening –> underestimation of parameter sensitivity –> very low cost
- OVAT analysis –> directionality –> better estimates of physical bounds of system response than OVAT screening
- OVAT analysis –> may still underestimate physical bounds –> curse of dimensionality –> no account of parameter interactions
- GSA –> quantitative –> accounts for parameter interactions
- GSA –> no account of directionality –> high computational cost
- For effective decision making, local AND global
- Spatio-temporal sensitivity maps –> sensor locations and monitoring intervals
- Sensitivity analysis –> analytical model –> process couplings
- Sensitivity analysis –> surrogate + numerical model –> constitutive couplings

ACCEPTED/PUBLISHED

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