

1. (a) Explain what are the physical quantities connected to the quantum numbers n , l , and m . Which values are possible for these quantum numbers?
- (b) How many degenerate levels exist for a hydrogen atom with principal quantum number $n = 4$? Justify your answer.
- (c) Which is the maximum number of electrons that can fill up an $4d$ orbital?
- (d) Give the selection rules for dipole transitions in atoms! How many steps are required for the decay from the excited state $n = 3$, $l = 2$ of the hydrogen atom to the ground state?

2. A particle has orbital angular momentum given by the quantum number $l = 2$ and spin angular momentum given by the quantum number $s = \frac{1}{2}$.
 - (a) How many distinct states are there with different values for the z components of the orbital and spin angular momenta?
 - (b) What are the possible values for the quantum number j that describes the total angular momentum of the particle?
 - (c) How many distinct states are there with different values for the magnitude and z component of total angular momentum?

3. (a) Write down the stationary Schrödinger equation of a particle of mass m for the potential $V(x) = 2x^2$. Simplify this using $m = 1$ and $\hbar = 1$
- (b) For which energy E is the function $\psi(x) = x \cdot e^{-x^2}$ a stationary wave function?

4. In a region of space, a particle with mass m and with zero energy has a stationary wave function $\psi(x) = 2b \cdot x \cdot e^{-x^2/a^2}$ where a and b are constants. Determine the potential energy $U(x)$ of the particle.

5. The spin operator is defined as: $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$, using Pauli matrices: $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$
 - (a) Write down the Pauli matrices: σ_x , σ_y , and σ_z
 - (b) Calculate $[S_z, S_x]$.
 - (c) Determine the eigenvalues and the eigenvectors of S_z .