

1. (a) Write down for the following 1-dimensional potential functions $V(x)$ the stationary Schrödinger equation for the corresponding wave functions $\psi_n(x)$ of a particle of mass m . Simplify this using $m = 1$ and $\hbar = 1$.
- (b) Sketch the potential together with the corresponding wave functions $\psi_n(x)$ of the ground states and of the first two excited states.
- (c) What can be said about the parity (symmetry) and the number of the nodes of the wave functions? How do these depend on the symmetry of the potential?
- (d) What are the expectation values for coordinate x and momentum p_x of a particle in the three sketched states? (without calculation)
- (e) Use for the discussions above the following potentials:

$V(x)$	$\psi(x)$
1) $2x^2$	$e^{-x^2} : xe^{-x^2} : (1-2x^2)e^{-x^2} : (1-4x^2)e^{-x^2}$
2) $\frac{9}{2}(x-1)^2$	$e^{-\frac{3}{2}(x-1)^2} : (x-1)e^{-\frac{3}{2}(x-1)^2} : (1-6(x-1)^2)e^{-\frac{3}{2}(x-1)^2}$
3) $\frac{1}{\cos^2(x)}$	$\cos^2(x) : \sin(x)\cos^2(x) : \sin^2(x)\cos^2(x)$

and check which of the accompanying functions, are solutions of the corresponding stationary Schrödinger equation. ($m = 1$ and $\hbar = 1$)

- (f) For the eigen functions found above, give the energy values and the time dependence of the complete wave function $\Psi(x, t)$!

The results for (e) and (f) (X means no eigen function and therefore no energy eigen value):

$$\begin{array}{l}
 V_1 : \quad 1 \quad 3 \quad \text{X} \quad 5 \\
 V_2 : \quad \frac{3}{2} \quad \frac{9}{2} \quad \frac{15}{2}
 \end{array}$$

$$V_3 : \quad 2 \quad \frac{9}{2} \quad \text{X} \qquad \Psi_n(x, t) = \psi_n(x) e^{-i \frac{E_n}{\hbar} t} = \psi_n(x) e^{-i E_n t} \quad (\hbar = 1)$$