1. Check the following operators regarding their properties: linearity, hermiticity (antihermiticity), and unitarity when applied on the function $\psi(x)$!

$$a)\ 3i\frac{d^{\,3}}{dx^3}\psi(x)\quad b)\ e^{\psi(x)}\quad c)\ \Big\{-\frac{d^{\,2}}{dx^2}+\sin(x)\Big\}\psi(x)\quad d)\ x\frac{d^{\,2}\psi(x)}{dx^2}$$

$$e)\ \psi(x)\frac{d\psi(x)}{dx}\quad f)\ 2i\frac{d\psi(x)}{dx}+3\quad g)\ 3i\frac{d\psi(x)}{dx}\quad h)\ \Big\{\frac{d^{\,2}}{dx^2}+2x\Big\}\psi(x)$$

$$i)\ exp\Big(i\,\frac{d}{dx}\Big)\psi(x)\quad j)\ exp\Big(\frac{d}{dx}\Big)\ \psi(x)\quad k)\ \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{ikx}\psi(k)dk\quad l)\ \psi^*(x)$$

$$m)\ \frac{1}{\psi(x)}\quad n)\ \left(x^2+i\frac{d}{dx}\right)\psi(x)\quad o)\ \psi(x+10)\cdot\psi(x)\quad p)\ \frac{d\psi(x)}{dx}$$

Results:
Not linear: b, e, f, l, m, o
Hermitian: a, c, g, h, i, n   Antihermitian: p
Not hermitian: d, j, k   Unitary: j, k

2. A commutator is the difference operator of a product of two operators in different order: $[\hat{A},\hat{B}]=\hat{A}\hat{B}-\hat{B}\hat{A}$
Show that products of operators can be expressed as:
$[\hat{A}\hat{B},\hat{C}]=\hat{A}[\hat{B},\hat{C}]+[\hat{A},\hat{C}]\hat{B}$ or $[\hat{A},\hat{B}\hat{C}]=\hat{B}[\hat{A},\hat{C}]+[\hat{A},\hat{B}]\hat{C}$
The elementary commutators of the coordinates and of the components of momentum are considered as basic knowledge. (The indices l und $k=1,2,3$ written in the tensorial way, represent the Cartesian coordinates x,y,z).
Show that: $\quad [\hat{x}_k,\hat{x}_l]=0,\quad [\hat{p}_k,\hat{p}_l]=0,\quad [\hat{p}_k,\hat{x}_l]=\frac{\hbar}{i}\ \delta_{kl}=-i\hbar\ \delta_{kl},$
with $\delta_{kl}=$ Kronecker symbol.

3. Check the results of the following commutators:
(The operator notation ˆ is omitted for the coordinates, because the operator $\hat{x}$ is identical with the value x of the coordinates. $\hat{L}_k$ are the components of the angular momentum.)

$$[\hat{p}_z,z^3]=3z^2\frac{\hbar}{i}\ ,\quad [\hat{p}_z,xy^2]=0\ ,\quad [\hat{p_x}^2,\frac{1}{x}]=-\frac{2\hbar^2}{x^3}+\frac{2i\hbar}{x^2}\hat{p}_x\ ,\quad [\hat{p}_x,e^{ix}]=\hbar e^{ix}$$

$$[\hat{p}_x,f(x)]=-i\hbar f'(x),\quad [y,\hat{p}_y^2]=2i\hbar\hat{p}_y,\quad [\hat{p}_y,\hat{L}_z]=i\hbar\hat{p}_x,\quad [\hat{p}_x,\hat{L}_z]=-i\hbar\hat{p}_y,$$

$$[\hat{L}_z,z]=0,\quad [\hat{L}_z,y]=-i\hbar x,\quad [\hat{L}_z,x]=i\hbar y,\quad [\hat{L}_x,\hat{\vec{L}}^2]=[\hat{L}_y,\hat{\vec{L}}^2]=[\hat{L}_z,\hat{\vec{L}}^2]=0,$$

$$[\hat{L}_z,\hat{L}_x]=i\hbar\hat{L}_y,\quad [\hat{L}_x,\hat{L}_y]=i\hbar\hat{L}_z,\quad [\hat{L}_y,\hat{L}_z]=i\hbar\hat{L}_x,\quad [\hat{p}_z,x^2+y^2+z^2]=-2i\hbar z.$$