1. Determine the normalized eigenfunctions $\psi_n(x)$ of a quantum particle of mass m in a symmetrical potential box with infinite walls at $x = \pm\frac{a}{2}$! Calculate for this case the expectation values $< x >, < p_x >, < x^2 >, < {p_x}^2 >$, the uncertainties Δx and Δp_x, and the probability current $j_x = \frac{\hbar}{2mi}(\psi^* \frac{d\psi}{dx} - \psi \frac{d\psi^*}{dx})$! Discuss the results!

2. Which of the functions: $\sin(\frac{\pi}{2a}x)$, $\sin(\frac{\pi}{a}x)$ and $\sin(\frac{2\pi}{a}x)$ are stationary eigenfunctions at $x = 0$ and $x = a$ of a quantum particle of mass m in a potential box with infinite walls? (Check the boundary conditions!)
   Determine for the eigenfunction the eigenvalue of the energy E_n, the normalization constants and the expectation values $< x >$ und $< p_x >$!
   Why is the normalized function:
   $$\psi(x,t) = \frac{1}{\sqrt{a}} \sin(\frac{\pi}{a}x) e^{-i\frac{E_1 t}{\hbar}} + \frac{1}{\sqrt{a}} \sin(\frac{2\pi}{a}x) e^{-i\frac{E_2 t}{\hbar}}$$
   not a stationary solution of the Schrödinger equation? What are the probabilities $W(E_1)$ and $W(E_2)$ when measuring the energy of a state ψ and what is the expectation value $< E >_\psi$?

3. Calculate for a quantum particle of mass m the stationary wave functions with negative energies ($E_n < 0$) in a finite symmetrical potential well with $V(x) = 0$ outside ($|x| > a$) and $V(x) = -V_0$ inside ($|x| < a$) the well. Find the general solution in the three potential ranges: left, middle, and right and give the boundary conditions at the edges of the well. Which other conditions are necessary for the determination of all 6 integration constants? How can the secular equation (characteristic equation) for the calculation of the energy values E_n be obtained? How does the problem simplifies by using the symmetry of the potential? How does the particle behavior change in the non-bonding states($E > 0$)?