1. Calculate the following integrals using the Dirac Delta Function $\delta(x)$:

$$I_1 = \int_0^2 \delta(x^2 - x - 1)dx$$

$$I_2 = \int_{-30}^{30} \delta(\cos x)dx$$

$$I_3 = \int_0^{\pi} \delta(\cos x)e^{ix}dx$$

$$I_4 = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \delta(\sin x)e^{ix}dx \qquad \text{and} \qquad I_5 = \int_0^2 \delta(x^2 - 1)(3x^2 - 1)dx$$

2. Calculate for the Gauß and for the Lorentz distribution functions of random variable $x$, the expectation values and the standard deviation.

   Gauß $\qquad w_G(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x - x_0)^2}{2\sigma^2}\right)$

   Lorentz(Cauchy) $\qquad w_L(x) = \frac{1}{\pi} \frac{\sigma}{(x - x_0)^2 + \sigma^2}$

   Check if the given distribution functions can be normalized. What is the full width at half maximum $x_{1/2}$ of the two functions?

3. Calculate for the Maxwell velocity distribution at given temperature $T$ $w(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right)$ the average of the squared velocity $< v^2 >$ and of the kinetic energy $< \varepsilon_{kin} >$ of a molecule of mass $m$!

4. What are the expectation value and the standard deviation for an ordinary ideal dice (perfect hexahedron)? What are the values when dicing with a regular octahedron (composed of 8 regular triangular faces) and a regular dodecahedron (composed of 12 regular pentagonal faces) on which the numbers from 1 to 8 and from 1 to 12 are marked respectively?