

1. Calculate the gradients for the followings scalar fields (potentials):

$$U(x, y, z) = \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25}$$

$$V(x, y, z) = \frac{x^3 - 12x}{16} + y^2 + z^2$$

$$\Psi(x, y, z) = x^2 + y^2$$

$$\Phi(\vec{r}) = 1/r \quad (r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2})$$

$$\Lambda(\vec{r}) = \vec{c} \cdot \vec{r}$$

Sketch the equipotential surfaces.

2. Determine the results of the superposition of two one-dimensional plane waves $A \sin(kx - \omega t)$ and $A \sin(kx + \omega t)$ having the same angular frequency ω , wavenumber k and amplitude A . What is changing when the frequency and the wavenumber of the two waves are slightly different? (standing waves, beats)
3. Two one dimensional waves having the same frequency ω and amplitude $A = 9.5$ cm which are propagating in the same direction are interfering having a phase difference $\Phi = 110^\circ$. Calculate the frequency and the amplitude of the resulting wave! How large should be the phase difference between the two waves, so that the resulting wave has the same amplitude as the initial waves?
4. Show that $\psi(\vec{r}, t) = \frac{1}{r} e^{i(kr - \omega t)}$ is a solution of the three dimensional wave equation. How does the wavefronts look like?
5. Two coherent light ($\lambda = 590$ nm) plane waves having the same amplitude interfere. Their wavefronts are 5 degrees tilted with respect to each other. Determine the distance between the interference minima (or maxima) in a plane perpendicular to the bisectors of the angle formed by the directions of the propagation of the two waves!
6. Find the form of the wave packet which results from interference of one dimensional plane waves having continuous wavenumber k in the $[k_0 - \frac{\delta k}{2}, k_0 + \frac{\delta k}{2}]$ spectral range, at given amplitude function $A(k) = 1$ and knowing the dispersion relation $\omega(k) = c \cdot k$ (c constant). What is the group velocity of the wave packet?