

4. EXERCISE

Main focus: Distribution Functions

Normalisation

Expectation values

Partial differential equations

Standard deviation

1. Normalise the following probability densities:

- (a) Gauss function

$$w_G(x) = \exp\left(-\frac{1}{2} \frac{(x - \mu)^2}{\sigma^2}\right) \quad (1)$$

- (b) Cauchy-Lorentz function

$$w_L(x) = \frac{\sigma}{x^2 + \sigma^2} \quad (2)$$

Calculate for Gauss and Lorentz distribution functions of random variable x , the expectation values and the standard deviations.

2. The Maxwell-Boltzmann distribution $w(v)$ for the speed of particles of mass m in gases at Temperature T is a statistic distribution in classical physics.

$$w(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right) \quad (3)$$

(k is Boltzmann's constant). Verify that the distribution is normalized! Determine the average speed $\langle v \rangle$ and kinetic energy $\langle e_{kin} \rangle$ of a molecule.

3. An electron is confined in a molecule of length $L = 1$ nm. Calculate its ground state energy and the first excitation energy in the approximation of an infinitely strong binding potential. What is the probability P to find the electron in the range between $x = 0$ and $x = 0.2$ nm in the ground state?