Computer course

Winter semester 13/14

3. EXERCISE

1. Plot a one-dimensional plane wave $A\sin(kx-\omega t)$ as a function of coordinate x and time t (use Manipulate). (Consider for simplicity the amplitude A, angular frequency ω, and wavenumber k being 1.) What happens by the superposition with the plane wave $A\sin((k+\delta k)x-(\omega+\delta\omega)t)$? Consider small values for δk and $\delta\omega$ (by example 0.1, or 0.01). Visualize and discuss the results!

2. Plot the Gauß, Lorentz, and spherical Bessel (order 0) distribution functions of random variable x, in the range from -20 to 20, for the same width $\sigma = 2$ when they are centered at 0.

   Gauß $\qquad w_G(x) = \dfrac{1}{\sigma\sqrt{2\pi}}\exp\left(-\dfrac{(x-x_0)^2}{2\sigma^2}\right)$

   Lorentz(Cauchy) $\qquad w_L(x) = \dfrac{1}{\pi}\dfrac{\sigma}{(x-x_0)^2+\sigma^2}$

   spherical Bessel function (order 0) $\qquad j_0(x) = \dfrac{\sin\frac{x}{\sigma}}{\pi x}$

   Compare the results! What happens when the width will decrease. Using the Manipulate command realise interactive plots for all three functions when σ decreses from 0.1 to 0.01 in the x-range from -1 to 1.

3. Calculate the following integrals using the Dirac Delta Function $\delta(x)$:

   $\mathrm{I}_1 = \displaystyle\int_0^2 \delta(2x^2-2x-1)dx$

   $I_2 = \displaystyle\int_{-5}^5 \delta(2x^2-2x-1)x^2dx$

   $I_3 = \displaystyle\int_{-20}^{20} \delta(\cos x)dx$

   $I_4 = \displaystyle\int_0^{\pi} \delta(\cos x)e^{-x}dx$

   $I_5 = \displaystyle\int_{\frac{-5\pi}{2}}^{\frac{\pi}{2}} \delta(\sin x)e^{ix}dx$

   $I_6 = \displaystyle\int_0^3 \delta(\ln x)e^{-x}dx$

   $I_7 = \displaystyle\int_0^4 \delta(x^2-5)(3x^2-x-1)dx$