Computer course

Winter semester 13/14

# 2. EXERCISE

Main focus: Vector fields
Differential Equations
Standing waves

1. (a) Plot the vector field $\vec{K}(\vec{r}) = (3x^2y, yz^2, -xz)$. Calculate the divergence and the Curl of this vector field.
   (b) Determine for the given potentials $U(x, y, z) = x^2 + y^2$ and $\Phi(\vec{r}) = \sin r$ (with $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$) the corresponding force fields $\vec{F_U}$ and $\vec{F_\Phi}$!
   (c) Calculate the source and circulation densities for these force fields.
   (d) Plot the vector fields.
2. Solve the differential equations:
   (a) Solve the differential equation for a one-dimensional frictionless harmonic oscillator in the x-direction. Consider the mass $m = 1$, the force constant (spring constant) of the restoring force $k = 10$, and the initial conditions $x(0) = 1$ and $\dot{x}(0) = 1$. Plot the solution as a function of time.
   (b) Consider now a damped oscillator, where a frictional force proportional to velocity is also acting. Solve the differential equation for a frictional coefficient $\gamma = 2$, and the same initial conditions as for $a$). Plot the solution as a function of time.
   (c) Consider now for the damped oscillator from $b$), also a periodic driving force $sin(5t)$. Solve the corresponding differential equation with the same initial conditions as at $a$). Plot the solution as a function of time.
   (d) Compare graphically the three solutions for $a$), $b$) and $c$). Discuss the results!
3. Plot a one-dimensional plane wave $A\sin(kx + \omega t)$ as a function of coordinate $x$ and time $t$. (Consider for simplicity the amplitude $A$, angular frequency $\omega$, and wavenumber $k$ being 1.) What happen by the superposition with the plane wave $A\sin(kx - \omega t)$? Visualize and discuss the results!

*Comment:* It is often necessary to restart Mathematica with all parameters and variables (**Evaluation > Quit Kernel > Local**). The command `Clear["Global‘*"]` clears the program of all definitions of the current session. The option `PlotRange` is often usefull to have a better understanding of the functions in a plot.