I. INTRODUCTION

A unique ESOP representation of Boolean functions \( f(x) \), called Specialized Normal From (SNF), was suggested in [2]. The number of cubes in the SNF \( f(x) \), expressed by the norm \(|\text{SNF}(f(x))|\), is a simple measure of the complexity of Boolean functions. In [1] was shown that only the most complex Boolean functions belong to the set of functions with a maximal value of \(|\text{SNF}(f(x))|\).

There are two drawbacks in using \(|\text{SNF}(f(x))|\) as a complexity measure. First, the calculation based on the definition of the SNF requires much memory for the expansion operation. This drawback may be eliminated by alternative approaches for their calculation. The second drawback is the property of the SNF that only a small number of different values \(|\text{SNF}(f(x))|\) exists. E.g., in case of \( n = 4 \), the 65,536 functions of \( \mathbb{B}^4 \) are split into 16 sets with regard to different values of \(|\text{SNF}(f(x))|\).

II. COMPLEXITY MEASURED BY DERIVATIVES

Generally, the complexity of a function increases with the number of variables the function is depending on. A function is independent of \( x_i \) if

\[
\frac{\partial f(x)}{\partial x_i} = 0.
\]

The number of function values 1 of \( \frac{\partial f(x)}{\partial x_i} \) indicates how much the function \( f(x) \) depends on \( x_i \). This degree of dependence \( \left| \frac{\partial f(x)}{\partial x_i} \right| \) can be used as a measure of the complexity.

A function is linear in \( x_i \) if

\[
\frac{\partial f(x)}{\partial x_i} = 1.
\]

In this case the variable \( x_i \) can be separated using an EXOR operation:

\[
f(x_i, x_1) = x_i \oplus f(x_1) .
\]

Hence, such a function is only slightly more complex than a function \( f(x_1) \). For that reason we use as complexity measure of a Boolean function \( f(x) = f(x_0, x_1) \)

\[
\text{cm}(x_0) = \begin{cases} 
2 \times n_1 \\ 2^n \times n_1 \times (\frac{1}{2} - 1) + (2 - \frac{1}{2}) \times 2^n
\end{cases}
\]

if \( n_1 \leq 2^{n-1} \)

otherwise

\[
\text{cm}(x_0) = \begin{cases} 
2 \times n_1 \\ 2^n \times n_1 \times (\frac{1}{2} - 1) + (2 - \frac{1}{2}) \times 2^n
\end{cases}
\]

where

\[
n_1 = \left| \frac{\partial f(x_0, x_1)}{\partial x_0} \right| .
\]

REFERENCES
