

## Lösungen zum Übungsblatt Komplexe Zahlen für Naturwissenschaftler (HM1)

1)  $z_1 + z_2 = -2 - 3i$ ,  $z_1 - z_2 = 2 + 5i$ ,  $z_1 \cdot z_2 = 4 - 2i$ ,  $\frac{z_1}{z_2} = -\frac{1}{5} - \frac{1}{10}i$ ,  $\overline{z_2} \cdot z_1 = -4 - 2i$ ,

$$\frac{\overline{z_2} \cdot z_2}{z_1} = -20i$$

2) a) w. A. b) f. A. c) w. A.

d)  $\sin \varphi \leq \sqrt{\sin^2 \varphi + \cos^2 \varphi} \rightarrow \sin \varphi \leq 1$  w. A. e)  $(1-i)^4 = -4$ , also f. A.

f)  $\sqrt{21+25} < \sqrt{49+9}$  w. A.

3) a)  $\operatorname{Re}(z) = \frac{3}{5}$ ,  $\operatorname{Im}(z) = -\frac{6}{5}$ , b)  $\operatorname{Re}(z) = -8$ ,  $\operatorname{Im}(z) = 0$ , c)  $\operatorname{Re}(z) = -\frac{3}{2}\sqrt{3}$ ,  $\operatorname{Im}(z) = \frac{3}{2}$

4) a)  $z = i \rightarrow r = 1$ ,  $\varphi = \frac{\pi}{2}$ ,  $z = (\cos \frac{\pi}{2} + i \sin \frac{\pi}{2})$

b)  $z = -1 - 2i \rightarrow r = \sqrt{5}$ ,  $\varphi \approx 243,43^\circ \hat{=} 4,25$ ,  $z = \sqrt{5}(\cos 243,43^\circ + i \sin 243,43^\circ)$

c)  $z = -1 - i \rightarrow r = \sqrt{2}$ ,  $\varphi = 225^\circ \hat{=} \frac{5}{4}\pi$ ,  $z = \sqrt{2}(\cos \frac{5}{4}\pi + i \sin \frac{5}{4}\pi)$

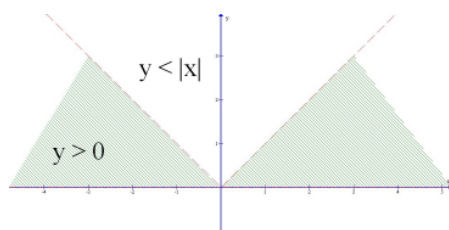
5) a)  $(2i - \sqrt{3})^8 = \sqrt{7}^8 (\cos 130,89^\circ + i \sin 130,89^\circ)^8 = 2401(\cos 327,15^\circ + i \sin 327,15^\circ)$   
 $= 2401(\cos 5,71 + i \sin 5,71) = 2017 - 1302,5i$

b)  $(-\frac{3}{2} - \frac{\sqrt{3}}{2}i)^{10} = \sqrt{3}^{10} (\cos 210^\circ + i \sin 210^\circ)^{10} = 243(\cos 300^\circ + i \sin 300^\circ)$   
 $= 243(\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}) = 243(\frac{1}{2} - \frac{\sqrt{3}}{2}i)$

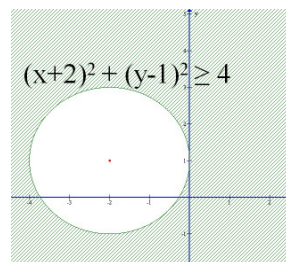
c)  $e^2 e^{-\frac{\pi}{3}i} = e^2 (\cos \frac{\pi}{3} - i \sin \frac{\pi}{3}) = e^2 (\frac{1}{2} - \frac{\sqrt{3}}{2}i)$

6) a)  $|z| = 2$ ,  $\arg(z) = 2\varphi - (-\varphi) = 3\varphi$ , b)  $|z| = 3$ ,  $\arg(z) = -\varphi + 3\varphi = 2\varphi$

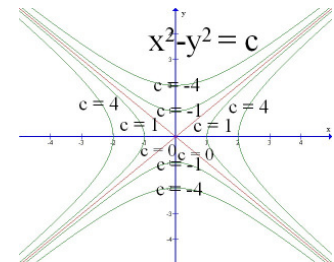
7) a)



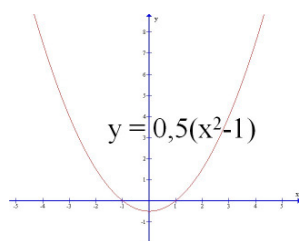
b)



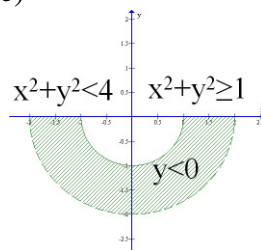
c)



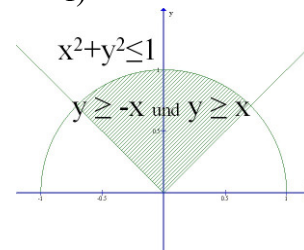
d)



e)



f)



8) a)  $z^3 = \cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi$ ,  $z_1 = e^{\frac{\pi}{2}i} = i$ ,  $z_2 = e^{\frac{7}{6}\pi i} = -\frac{1}{2}\sqrt{3} - \frac{1}{2}i$ ,  $z_3 = e^{\frac{11}{6}\pi i} = \frac{1}{2}\sqrt{3} - \frac{1}{2}i$

b)  $z^5 = \sqrt{89}(\cos 57,99^\circ + i \sin 57,99^\circ)$ ,  $z_k = \sqrt[5]{89}e^{\frac{1}{5}(57,99^\circ + k360^\circ)}$ ,  $k = 0, 1, 2, 3, 4$

$z_1 = 1,53 + 0,315i$ ,  $z_2 = 1,75 + 1,56i$ ,  $z_3 = -1,43 + 0,65i$ ,  $z_4 = -1,06 - 1,16i$ ,  $z_5 = 0,77 - 1,36i$

- 9) a)  $z_1 = 1$ ,  $z_2 = \frac{1}{2}(1 + \sqrt{3}i)$ ,  $z_3 = \frac{1}{2}(-1 + \sqrt{3}i)$ ,  $z_4 = -1$ ,  $z_5 = -\frac{1}{2}(1 + \sqrt{3}i)$ ,  $z_6 = \frac{1}{2}(1 - \sqrt{3}i)$
- b)  $z_1 = \cos 15^\circ - i \sin 15^\circ = 0,96 - 0,26i$ ,  $z_2 = \cos 105^\circ - i \sin 105^\circ = -0,26 - 0,96i$   
 $z_3 = \cos 195^\circ - i \sin 195^\circ = -0,96 + 0,26i$ ,  $z_4 = \cos 285^\circ - i \sin 285^\circ = 0,26 + 0,96i$
- c)  $z^5 = \sqrt{125}(\cos 153,4^\circ + i \sin 153,4^\circ)$ ,  $z_k = \sqrt[5]{125}e^{\frac{1}{5}(153,4^\circ + k360^\circ)}$ ,  $k = 0,1,2,3,4$   
 $z_1 = 1,39 + 0,83i$ ,  $z_2 = -0,36 + 1,58i$ ,  $z_3 = -1,61 + 0,15i$ ,  $z_4 = -0,64 - 1,49i$ ,  $z_5 = 1,22 - 1,07i$
- d)  $z^2(1+i) + 2z = z(z(1+i) + 2) = 0$ ,  $z_1 = 0$ ,  $z_2 = -\frac{2}{1+i} = -1+i$