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FUZZY BILINEAR BILEVEL PROBLEMS

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ABSTRACT. In the present paper a global optimal solution of a fuzzy bilevel optimization problem is found. The fuzzy optimization problem supposed to be bilinear. The upper and lower level constraint sets are defined as fuzzy and crisp polytopes, respectively. The lower level fuzzy optimization problem is described as a set of Pareto optimal solutions of a corresponding biobjective optimization problem. The preferable optimal solution on each level-cut supposed to have a maximal membership function value, i.e. the solution has the highest potential being realized by the lower level decision-maker. With this solution the upper level fuzzy optimization problem is solved with the use of a stability region. Because of the use of the membership function values, using this approach a decision-maker can see a correlation among solutions and quantitatively measure the advantage of one choice over others.

1. Introduction

Bilevel programming problems are challenging problems of mathematical optimization, which are interesting from the theoretical point-of-view (since it is a special case in nonsmooth optimization) and have a variety of applications. They are hierarchical problems of two decision makers, in which one - the so-called leader - has the first choice and the other one - the so-called follower - reacts optimally on the leader’s selection. The formulation of the bilevel programming problem for crisp (i.e. known and fixed) data can be found e.g. in Dempe (2002) and is formulated in the present paper for fuzzy data.

Since its first formulation by H. v. Stackelberg in market economy, bilevel optimization has successfully been applied to many real world problems Dempe (2002); Fortuny-Amat and McCarl (1981); Parraga (1981); Bard et al. (1998); Björndal and Jörnsten (2005); Camacho (2006); Hobbs and Nelson (1992); Marcotte and Savard (2001).

Considering the inherently difficult nature of bilevel problems due to their nonconvexity, nonsmoothness and implicitly determined feasible set, it is difficult to design convergent algorithms, and the few algorithms that converge appear to be very slow most of the time. The
reason that a global optimal solution of the bilevel optimization problem is difficult to compute is due to $\mathcal{NP}$-hardness of this problem (see Ben-Ayed and Blair (1998); Blair (1992)).

If the data involved in the bilevel optimization problem are only approximately known, fuzzy bilevel optimization problem arises. This theory proved very useful in many applied sciences, such as economics, physics, etc. Budnitzki (2013); Dempe et al. (2009); Dempe and Starostina (2006); Liu and Kao (2004); Peidro et al. (2010); Ruziyeva (2013); Wu and Xu (2008); Zhang et al. (2010). But whereas very important for number of applications, this problem is poorly investigated. While some convergent algorithms for crisp bilevel problems already exist in the literature Bard (1982); Önal (1993); Tuy and Ghanadan (1998); Wu et al. (1998), solution strategies for fuzzy bilevel programming problems are an emerging new field with a wide range of practical applicability.

In the present paper an effective algorithm for computation a global optimal solution for the upper level decision-maker is developed under assumption that the lower level decision-maker chooses the optimal solution as well.

The algorithm is based on the membership function approach Dempe and Ruziyeva (2012). The lower level fuzzy optimization problem is solved by a method of level-cuts (see Dempe and Ruziyeva (2011); Ruziyeva and Dempe (2012) for nonlinear case). Elements of the Pareto set of the corresponding biobjective optimization problem are interpreted as potential optimal solutions of the lower level fuzzy optimization problem on a certain level-cut. The optimal solution is selected due to the highest membership function value.

Then, this solution is used on the upper level such that, with respect to its region of stability, the optimal solution of the leader is found. Comparing all optimal solutions with respect to the upper level function value, the global optimal solution is found.

The roadmap of the present paper is the following. The notions, that we use in the present paper are given in Section 2. The fuzzy bilevel optimization problem is formulated in Section 3. The solution technique is described in Section 4. The paper is concluded with an illustrative example in Section 5.

2. Preliminaries

2.1. Basic notions.

Definition 1. A fuzzy set $\tilde{C}$ is defined as a pair $(C, \mu_{\tilde{C}})$, where $C$ is a crisp set ($C \subset \mathbb{R}^n$) and $\mu_{\tilde{C}} : C \rightarrow [0, 1]$ is the membership function of
the fuzzy set \( \tilde{C} \). For each element \( x \in C \), the value of \( \mu_{\tilde{C}}(x) \) is called the grade of membership of \( x \) in \( \tilde{C} \).

**Corollary 1.** The empty fuzzy set \( \tilde{D} \) is defined with its membership function \( \mu_{\tilde{D}}(x) = 0 \) for all \( x \in D \), i.e. is the same as a crisp empty set.

A fuzzy number \( \tilde{c} \) is an element of the nonempty fuzzy set \( \tilde{C} \) enriched with the nontrivial membership function \( \mu_{\tilde{c}} \). In other words, a real fuzzy number is a convex continuous fuzzy subset of a real line. More precisely, a fuzzy number was defined by Dubois and Prade (1978) as follows:

**Definition 2.** A real fuzzy number \( \tilde{n} \) is a convex continuous fuzzy subset of the real line, whose membership function \( \mu_{\tilde{n}} \) is

- a continuous mapping from \( \mathbb{R} \) to the closed interval \([0, 1] \);
- constant on \( (-\infty, c] \) : \( \mu_{\tilde{n}}(x) = 0 \) \( \forall x \in (-\infty, c] \);
- strictly increasing on \([c, a] \);
- constant on \([a, b] \) : \( \mu_{\tilde{n}}(x) = 1 \) \( \forall x \in [a, b] \);
- strictly decreasing on \([b, d] \);
- constant on \([d, +\infty) \) : \( \mu_{\tilde{n}}(x) = 0 \) \( \forall x \in [d, +\infty) \).

Here \( a, b, c \) and \( d \) are real numbers.

The simplest way to define a fuzzy number \( \tilde{n} \) is presented in e.g. Buckley (1995) as follows.

**Definition 3.** A continuous triangular fuzzy number \( \tilde{n} \) is represented with a triple \((n_L, n_T, n_R)\), where \( n_L < n_T < n_R \) and the membership function \( \mu_{\tilde{n}} \) is piecewise-linear.

A possible membership function of triangular fuzzy number \( \tilde{n} \) is presented in Fig. 1. We interpret these three numbers as \( n_T \) - a best estimation, \( n_L \) - a minimum possible value, \( n_R \) - a maximum possible value.

**Definition 4.** A normalized fuzzy number is a fuzzy number with a membership function, that reaches value equal to 1.

We consider in the present paper only normalized fuzzy numbers.

**Definition 5.** A level-cut (\( \alpha \)-cut, \( \alpha \)-level) of a fuzzy number \( \tilde{c} \) is a special threshold described as an interval \([c_L(\alpha), c_R(\alpha)] \subset \mathbb{R} \) for some fixed \( \alpha \in [0, 1] \) (see Fig. 2). Here \( c_L(\alpha) \) and \( c_R(\alpha) \) represent left- and right-hand side bounds of the fuzzy number \( \tilde{c} \) on this certain \( \alpha \)-cut.

Let us extend the concept of a fuzzy set to a space of fuzzy numbers and a fuzzy number to a fuzzy vector.
Definition 6. A space of fuzzy numbers $\mathfrak{F}^n$ is defined as a pair $(\mathbb{R}^n, \mu)$, where $\mu : \mathbb{R}^n \to [0, 1]$ is a membership function of the elements of the fuzzy space $\mathfrak{F}^n$.

Definition 7. A fuzzy vector $\tilde{c}$ is an element of the fuzzy space $\mathfrak{F}^n$. Each component of the fuzzy vector is enriched with a nontrivial membership function.

2.2. Operations.

Proposition 1. Two fuzzy vectors are equal if and only if (iff) they have the same membership functions, i.e. for $\tilde{a}, \tilde{b} \in \mathfrak{F}^n$

$$\tilde{a} = \tilde{b} \iff \mu_{\tilde{a}}(x) = \mu_{\tilde{b}}(x) \ \forall x \in \mathbb{R}^n.$$ 

Proposition 2. Let $\tilde{a}, \tilde{b} \in \mathfrak{F}^n$. Then the sum of two fuzzy vectors $\tilde{a} + \tilde{b}$ is defined as a fuzzy vector $\tilde{d} \in \mathfrak{F}^n$ with the following property

$$d_L(\alpha) = a_L(\alpha) + b_L(\alpha) \text{ and } d_R(\alpha) = a_R(\alpha) + b_R(\alpha) \text{ for all } \alpha \in [0, 1].$$
Here $a_L(\alpha)$ and $a_R(\alpha)$ are left- and right-side values of the fuzzy vector $\tilde{a}$ on a certain $\alpha$-cut. The same notation is used for the fuzzy vectors $\tilde{b}$ and $\tilde{d}$.

**Definition 8.** Wu (2008, 2009) Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy vectors in $\mathfrak{F}^n$. If there exists a fuzzy vector $\tilde{c} \in \mathfrak{F}^n$, such that $\tilde{c} + \tilde{b} = \tilde{a}$ (note that fuzzy addition is commutative) and $\tilde{c}$ is unique, then $\tilde{c}$ is called the Hakuhara difference of $\tilde{a}$ and $\tilde{b}$ and is denoted by $\tilde{a} \ominus_H \tilde{b}$.

The following proposition follows from Proposition 2 immediately, and is very useful for the future considerations of the differentiation of fuzzy-valued functions.

**Proposition 3.** Let $\tilde{a}, \tilde{b} \in \mathfrak{F}^n$. If the Hakuhara difference $\tilde{c} = \tilde{a} \ominus_H \tilde{b}$ exists, $\tilde{c} \in \mathfrak{F}^n$, then

$$c_L(\alpha) = a_L(\alpha) - b_L(\alpha) \quad \text{and} \quad c_R(\alpha) = a_R(\alpha) - b_R(\alpha) \quad \forall \alpha \in [0, 1].$$

Here $[c_L(\alpha), c_R(\alpha)]$ is an $\alpha$-cut of the fuzzy vector $\tilde{c}$.

2.3. Fuzzy relations.

**Proposition 4.** A fuzzy number $\tilde{a}$ is not greater than a fuzzy number $\tilde{b}$ if the $\alpha$-levels of $\tilde{a}$ do not exceed the $\alpha$-levels of $\tilde{b}$, i.e.

$$\tilde{a} \preceq \tilde{b} \iff [a_L(\alpha), a_R(\alpha)] \preceq [b_L(\alpha), b_R(\alpha)] \quad \forall \alpha \in [0, 1].$$

**Definition 9.** The relation $[a, b] \preceq [c, d]$ holds true if $a \leq c$ and $b \leq d$ for $a, b, c, d \in \mathbb{R}$.

For fuzzy numbers $\tilde{a}$ and $\tilde{b}$ and crisp numbers $a$ and $b$ the following corollaries hold true.

**Corollary 2.** $\tilde{a} \preceq \tilde{b} \iff a_R(0) \leq b$.

**Corollary 3.** $a \preceq \tilde{b} \iff a \leq b_L(0)$.

**Proposition 5.** A fuzzy number $\tilde{a}$ is smaller than a fuzzy number $\tilde{b}$ if for all $\alpha \in [0, 1]$ the $\alpha$-levels of $\tilde{a}$ are smaller than the $\alpha$-levels of $\tilde{b}$, i.e.

$$\tilde{a} \prec \tilde{b} \iff [a_L(\alpha), a_R(\alpha)] \prec [b_L(\alpha), b_R(\alpha)] \quad \forall \alpha \in [0, 1].$$

**Definition 10.** Chanas and Kuchta (1996) The relation $[a, b] \prec [c, d]$ holds true if $a \leq c$ and $b \leq d$ (with at least one strong inequality) for $a, b, c, d \in \mathbb{R}$. 

3. Fuzzy bilevel bilinear optimization problem

Let us consider the following fuzzy bilinear bilevel optimization problem

\[
F(\tilde{c}, x) \rightarrow \min_{\tilde{c} \in \tilde{C}} \quad \text{s.t. } f(\tilde{c}, x) \rightarrow \min_{x \in X}.
\]

with an \(k\)- and \(n\)-dimensional vectors of decision variables \(\tilde{c}\) and \(x\), respectively, under the assumption that

- \(F(\tilde{c}, x)\) and \(f(\tilde{c}, x)\) are bilinear continuous functions;
- The follower’s feasible set \(X = \{x : Ax \leq b, x \geq 0\}\) is a crisp polytope, where
  - \(A \in \mathbb{R}^{m \times n}\) is the constraint matrix,
  - \(b \in \mathbb{R}^{m}\) is the right-hand side vector.
- The leader’s feasible set \(\tilde{C}\) is a fuzzy polytope defined through

**Definition 11.** A fuzzy polytope \(\tilde{C}\) is defined as a closed convex fuzzy set given by finite number of fuzzy inequalities, i.e.\n
\[\tilde{C} = \{\tilde{P}\tilde{c} \preceq \tilde{q}, 0 \preceq \tilde{c}\},\]

where \(\tilde{P} \in \mathbb{F}^{l \times k}\) is a fuzzy matrix, \(\tilde{q} \in \mathbb{F}^{l}\) is a fuzzy vector and the order relation is defined by Proposition 4.

That means that the lower level fuzzy optimization problem is stated for the fixed vector of coefficients \(\tilde{c}^\times \in \tilde{C}\) in a form of fuzzy linear optimization problem

\[
\tilde{c}^\times \top x \rightarrow \min_{x \in X}.
\]

This problem is solved with the approach based on minimization of the \(\alpha\)-cut on the feasible set. So the uncertainty in the fuzzy optimization problem is reflected through a set of optimal solutions, i.e. a set of Pareto optimal solutions of corresponding biobjective optimization problem. Under the assumption that this set consists of more than one element, the decision-maker can improve the choice relying on some criteria that are not a priori considered in the optimization problem. This solution method described in details in Ruziyeva and Dempe (2012); Dempe and Ruziyeva (2011). In linear case the feasible set is the polytope \(X\). Thus, a solution on each level-cut (one of the basic solutions) is one of the vertices of this set Dempe and Ruziyeva (2012).

In the present paper we describe a solution algorithm for fuzzy bilevel optimization problem (1) that supposes, that the membership function
values of the solutions of lower level fuzzy linear optimization problem (2) can be found. In Dempe and Ruziyeva (2012) the interested reader can find an algorithm for the computation of the membership function’s value for the wide class of triangular fuzzy numbers, that can easily be extended to the class of \( L - R \) fuzzy numbers.

Henceforth, we assume that the membership function values for all vertices of this polytope can be computed (for non-optimal solutions the membership function is zero). As soon as the solution of problem (2) is a fuzzy set of feasible points, elements of this set with the largest membership function values should be selected, since these have the largest potential of being realized Chanas and Kuchta (1994).

**Definition 12.** A fuzzy solution of problem (2) \( \hat{x} \) is a best solution provided that \( \mu(\hat{x}) \geq \mu(x_i(\tilde{c})) \) for all other fuzzy solutions \( x_i \) (\( i = 1, \ldots, N \)), where \( N \) is a number of basic solutions.

On the basis of aforesaid we suppose that the optimal solution of fuzzy linear optimization problem (2) \( \hat{x} := \hat{x}(\tilde{c}^\ast) \) for the fixed \( \tilde{c}^\ast \in \tilde{C} \) possess the maximal membership function value \( \mu(\hat{x}) \).

**Definition 13.** A region of stability of the fuzzy solution \( \hat{x} \) is

\[
\tilde{R}(\hat{x}) = \{ \tilde{c} : f(\tilde{c}, \hat{x}) \preceq f(\tilde{c}, x_i) \ \forall i \in B, \tilde{c} \in \tilde{C} \}
\]

where \( B \) is a set of basic indices of lower level fuzzy linear optimization problem (2).

**Theorem 1.** The region of stability is a fuzzy polytope.

*Proof.* In Definition 13 region of stability is defined as an intersection of a finite number of inequalities. Under assumption that the feasible set \( \tilde{C} \) is a fuzzy polytope, the region of stability is also a fuzzy polytope (see Definition 11).

We recall now

**Definition 14.** A connected set is a topological set that cannot be represented as a union of two or more disjoint nonempty open subsets.

**Corollary 4.** A region of stability is a convex connected set.

**Remark 1.** A region of stability is not a single point since the rows of the matrix \( A \) are linear independent. If a region of stability of some solution \( x_0 \in X \) is a point, then the solution \( x_0 \) is nonstable.

**Definition 15.** Interior of the fuzzy polytope \( \tilde{R}(\hat{x}) \) is a fuzzy open set

\[
\text{int}(R(\hat{x})) = \{ \tilde{c} : f(\tilde{c}, \hat{x}) \prec f(\tilde{c}, x_i) \ \forall i \in B, \tilde{c} \in \tilde{C} \},
\]

where the order relation is defined with Proposition 5.
Figure 3. $n$ regions of stability cover the feasible set of the leader

4. Algorithm

The idea of the algorithm is enumerative technique: we cover the feasible set of the leader with the regions of stability for the best lower level solutions under assumption, that regions of stability can be exactly calculated.

Set $k := 1$ and fix a random fuzzy vector $\tilde{c}_k \in \tilde{R} := \tilde{C}$.

STEP 1. For the fixed $\tilde{c}_k$ compute a best solution $\hat{x}_k := \hat{x}(\tilde{c}_k)$ of the fuzzy lower level problem (2).

STEP 2. Find a region of stability $\tilde{R}(\hat{x}_k)$ using Definition 13.

STEP 3. Solve the upper level fuzzy optimization problem

$$ (3) \quad F(\tilde{c}, \hat{x}_k) \rightarrow \min_{\tilde{c} \in \tilde{R}(\hat{x}_k)} $$

Denote an optimal solution of problem (3) through $\tilde{c}_k^*$. Retain the pair $(\tilde{c}_k^*, \hat{x}_k)$.

STEP 4. $\tilde{R} := \tilde{R} \setminus \tilde{R}(\hat{x}_k)$. If $\tilde{R} = \emptyset$, then STOP. Else fix $\tilde{c}_{k+1} \in int(\tilde{R})$ and go to STEP 1 with $k := k + 1$ (see Fig. 3).

STEP 5. Compare the pairs $(\tilde{c}_1^*, \hat{x}_1), \ldots, (\tilde{c}_n^*, \hat{x}_n)$, where $n$ is number of iterations, and choose $(\tilde{c}^*, \hat{x})$ - the best pair with respect to the upper level function $F(\tilde{c}, x)$. 
Remark 2. If there exist two different pairs \((\tilde{c}_i^*, \hat{x}_i)\) and \((\tilde{c}_j^*, \hat{x}_j)\) such that \((\tilde{c}_i^*, \hat{x}_i) \neq (\tilde{c}_j^*, \hat{x}_j)\) with the same upper level function value \(F(\tilde{c}_i^*, \hat{x}_i) = F(\tilde{c}_j^*, \hat{x}_j)\), we suggest to choose such a pair that has a best second component, i.e. if \(\mu(\hat{x}_i) > \mu(\hat{x}_j)\), then the pair \((\tilde{c}_i^*, \hat{x}_i)\) is better (more preferable) than \((\tilde{c}_j^*, \hat{x}_j)\).

Theorem 2. The algorithm is convergent.

Proof. Consider lower level problem (2). The feasible set \(X\) has a finite number of vertices, and we are interested only in \(N\) basic solutions. Clear, that \(N < \infty\). Thus, we obtain for each best solution on the lower level its region of stability. The total number of the regions of stability cannot exceed \(N\). Taking into consideration Corollary 4 and Remark 1, the theorem is proved.

Theorem 3. The pair \((\tilde{c}^*, \hat{x})\) is a global optimal solution of fuzzy bilevel programming problem (1).

Proof. The proof of this fact is obvious with the rule of contraries. Suppose that there exist other global optimum, e.g. the pair \((\tilde{c}_0, x_0)\) such that \(F(\tilde{c}_0, x_0) < F(\tilde{c}^*, \hat{x})\). In consideration of STEP 4 and its STOP criteria the region of stability \(R(x_0)\) of the solution \(x_0\) of the lower level problem is considered in one of the iterations. But the assumption of the theorem is following. Since after all the comparisons at the STEP 5 of the algorithm the best solution is \((\tilde{c}^*, \hat{x})\), consequently \(F(\tilde{c}^*, \hat{x}) < F(\tilde{c}_0, x_0)\). This is contradiction to the assumption that \((\tilde{c}_0, x_0)\) is global optimal solution.

As soon as the fuzzy numbers are non-comparable, the way of the computation of the region of stability according to Definition 13 in some cases can be too complicated.

There exist few heuristic ways to overcome this problem. We suggest to compute the region of stability using Yager-index within the following formula:

\[
R_I(\hat{x}) = \{\tilde{c} : I(\tilde{c}) \hat{x} \leq I(\tilde{c}) x_i \forall i \in B, \tilde{c} \in \tilde{C}\},
\]

where \(B\) is the set of basic indices and \(I(\tilde{c}) := \frac{1}{2} \int_0^1 [c_L(\alpha) + c_R(\alpha)] d\alpha\) is the Yager ranking index Liu and Kao (2004); Ruziyeva and Dempe (2013).

For the simplest case, that is presented in the next Section, we assume that the fuzzy numbers are presented by their triangular membership functions.
To accomplish the discussion it is interesting to explain the results by giving an example using triangular fuzzy numbers. As soon as this example is linear and not high dimensional, we compute regions of stability within Definition 13.

\[
F(\tilde{c}, x) = (x_1, x_2) \left( \begin{array}{c} \tilde{c}_1 \\ \tilde{c}_2 \end{array} \right) \min_{\tilde{c}} \quad
\begin{align*}
\tilde{c}_1 + \tilde{c}_2 & \preceq 6 \\
\tilde{4} & \preceq \tilde{c}_1 + \tilde{c}_2 \\
0 & \preceq \tilde{c}_1 \\
2 & \preceq \tilde{c}_2
\end{align*}
\]

where \( x \) solves

\[
f(\tilde{c}, x) = [ (\tilde{c}_1, \tilde{c}_2) + (1, 1) ] \left( \begin{array}{c} x_1 \\ x_2 \end{array} \right) \rightarrow \max_x \\
\text{s.t. } x_1 + 3x_2 & \geq 3 \\
2x_1 + x_2 & \leq 8 \\
x & \geq 0,
\]

where \( \tilde{4} = (3, 4, 5) \). The aim is to find the best optimal solution \( c^* \) for the leader. Let us assume that the fuzzy variable \( \tilde{c} \) is presented as a normalized fuzzy number.

Let us set \( \tilde{R} := \tilde{C} \) and fix a random fuzzy vector \( \tilde{c}^1 = \left( \begin{array}{c} 3 \\ 3 \end{array} \right) \).

STEP 1 (Iteration 1). A best solution of the fuzzy lower level problem (2) for this \( \tilde{c}^1 \) is \( \hat{x}^1 = \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \).

STEP 2 (Iteration 1). A region of stability \( \tilde{R}(\hat{x}^1) \) is computed as follows. \( \tilde{R}(\hat{x}^1) = \{ \tilde{c} : \tilde{c}_2 \preceq 3\tilde{c}_1 + 2, \tilde{c} \in \tilde{C} \} \).

STEP 3 (Iteration 1). Solve the upper level fuzzy optimization problem

\[
F(\tilde{c}, \hat{x}^1) \rightarrow \min_{\tilde{c}} \quad \tilde{c} \in \tilde{R}(\hat{x}^1).
\]

An optimal solution of this problem is \( \tilde{c}_1^* = \left( \begin{array}{c} \tilde{2} \\ 2 \end{array} \right) \), where \( \tilde{2} = (1, 2, 3) \).

Retain the pair \( (\tilde{c}_1^*, \hat{x}^1) \).

STEP 4 (Iteration 1). As soon as \( \tilde{R} := \tilde{R} \setminus \tilde{R}(\hat{x}^1) \neq \emptyset \), let us fix \( \tilde{c}^2 = \left( \begin{array}{c} 0 \\ 6 \end{array} \right) \in int(\tilde{R}) \) and go to STEP 1.
STEP 1 (Iteration 2). A best solution of the fuzzy lower level problem (2) for this $\tilde{c}^2$ is $\tilde{x}^2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

STEP 2 (Iteration 2). A region of stability is $\tilde{R}(\tilde{x}^1) = \{\tilde{c} : 3\tilde{c}_1 + 2 \preceq \tilde{c}_2, \tilde{c} \in \tilde{C} \}$.

STEP 3 (Iteration 2). Solve the upper level fuzzy optimization problem

$$F(\tilde{c}, \tilde{x}^2) \rightarrow \min_{\tilde{c} \in \tilde{R}(\tilde{x}^2)}$$

An optimal solution is $\tilde{c}^*_2 = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$. Retain the pair $$(\tilde{c}^*_2, \tilde{x}^2)$$. 

STEP 4 (Iteration 2). We stop the algorithm because $\tilde{R} = \emptyset$.

STEP 5. Compare the values of function $F(\tilde{c}^*_1, \tilde{x}^1) = 2$ and $F(\tilde{c}^*_2, \tilde{x}^2) = 0$. Clear, that a global optimal solution is $$(\tilde{c}^*_2, \tilde{x}^2)$$.

6. Conclusions

In the present paper the algorithm solving the fuzzy bilevel optimization problem is described for the bilinear case. At the single-level the fuzzy optimization problems are solved by methods of the level-cuts and the scalarization technique.

The optimal solution on the certain level is selected due to the highest membership function value as the best solution. Then, this solution is involved in a switch between upper and lower level problems such that, with response to the region of stability, the described algorithm gives a global optimal solution of the fuzzy bilinear bilevel optimization problem.

References


