5 Integralrechnung

Aufgabe 5.1.

a) $\displaystyle\int \frac{1}{1-\cos^2 x}\,dx = -\cot x + c$ b) $\displaystyle\int \tan^2 x\,dx = \tan x - x + c$

c) $\displaystyle\int \left(\frac{x+2}{x^2} + \frac{x^2}{x+2}\right) dx = \ln|x| - \frac{2}{x} + \frac{x^2}{2} - 2x + 4\ln|x+2| + c$

Aufgabe 5.2.

a) $\displaystyle\int e^{1-x}\,dx = -e^{1-x} + c$ b) $\displaystyle\int \sinh(1+2x)\,dx = \frac{1}{2}\cosh(1+2x) + c$

c) $\displaystyle\int \frac{3}{\cos^2(3x)}\,dx = \tan(3x) + c$ d) $\displaystyle\int \frac{5}{1+4x}\,dx = \frac{5}{4}\ln|4x+1| + c$

e) $\displaystyle\int \frac{5}{1+4x^2}\,dx = \frac{5}{2}\arctan(2x) + c$ f) $\displaystyle\int \frac{2}{\sqrt[5]{3-\frac{x}{4}}}\,dx = -10\sqrt[5]{\left(3-\frac{x}{4}\right)^4} + c$

g) $\displaystyle\int \left(3^{x-1}\right)^5\,dx = \frac{\left(3^{x-1}\right)^5}{5\ln 3} + c$

Aufgabe 5.3.

$$\int \frac{x}{\sqrt{x^2+8}}\,dx = \sqrt{x^2+8} + c$$

Aufgabe 5.4.

a) $\displaystyle\int \frac{x^2}{x^3-8}\,dx = \frac{1}{3}\ln|x^3-8| + c$ b) $\displaystyle\int \frac{1}{x\ln(2x)}\,dx = \ln|\ln(2x)| + c$

c) $\displaystyle\int \frac{\ln(x^5)}{x}\,dx = \frac{5}{2}\ln^2 x + c$

Aufgabe 5.5.

$$\int \frac{6-5x^2-2x^3}{(x^2+4x+4)(x^2+2x+2)}\,dx$$
$$= -\ln|x+2| - \frac{1}{x+2} - \frac{1}{2}\ln\left|x^2+2x+2\right| + 3\arctan(x+1) + c$$

Aufgabe 5.6.

$$\frac{3x^4}{x^4+5x^2+4} = 3 - \frac{15x^2+12}{(x^2+1)(x^2+4)}$$

Ansatz:

$$\frac{15x^2+12}{(x^2+4)(x^2+1)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4}$$

Aufgabe 5.7.

a) $$\int (x^2-4)\cos(2x)\,dx = \frac{1}{4}\left[\left(2x^2-9\right)\sin(2x)+2x\cos(2x)\right]+c$$

b) $$\int (x+1)\ln\left(\frac{x}{2}\right)\,dx = x\left(\frac{x}{2}+1\right)\ln\left(\frac{x}{2}\right)-\frac{x^2}{4}-x+c$$

c) $$\int \arctan x\,dx = x\arctan x-\frac{1}{2}\ln\left(x^2+1\right)+c$$

d) $$\int \sin^2 x\,dx = \frac{1}{2}\left(x-\frac{1}{2}\sin(2x)\right)+c$$

Aufgabe 5.8.

a) $$\int_0^1 x\sqrt{2-x^2}\,dx = \frac{1}{3}\left(2\sqrt{2}-1\right)$$

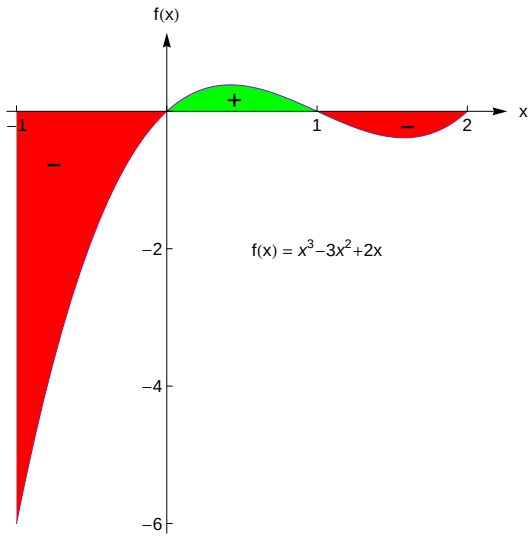
b) $$\int_1^3 \frac{|x-2|}{x^2}\,dx = \frac{2}{3}+\ln\left(\frac{3}{4}\right)$$

c) $$\int_{1/2}^{e/2} \frac{\ln(2x)}{x}\,dx = \frac{1}{2}$$

Aufgabe 5.9.

$$A = \frac{11}{4}$$

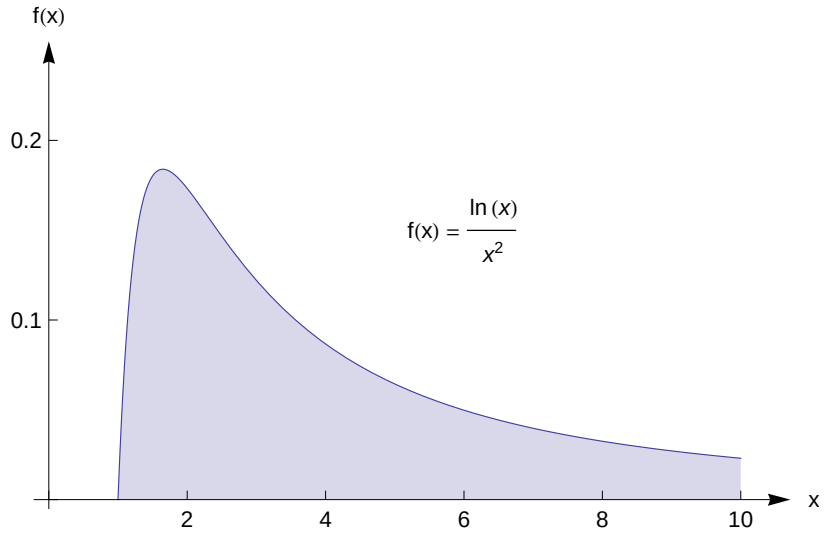
Grafische Veranschaulichung



Aufgabe 5.10.

$$\int_1^\infty \frac{\ln x}{x^2}\,dx = 1$$

Grafische Veranschaulichung



Aufgabe 5.11.

a) $\int_0^1 \ln x\,dx = -1$ b) $\int_0^1 \left(1 - \frac{1}{x}\right)\,dx = -\infty$