Numerical simulation of magnetotelluric fields at Stromboli

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SUMMARY

Stromboli is a small volcanic island in the Mediterranean Sea off the west coast of Italy. It is famous for its characteristic Strombolian eruptions. To get a better understanding of these processes further explorations of the inner structure of the volcano are essential. By carrying out numerical simulations, we aim at showing that the magnetotelluric method using a wide frequency range, e.g. $10^{-4}...10^{4}$ Hz, is applicable to this task.

To compute accurate electromagnetic fields the geometry of Stromboli volcano and the surrounding bathymetry need to be considered as detailed as possible. This becomes feasible using 2D and 3D finite element techniques on unstructured triangular and tetrahedral grids. First numerical simulations of MT measurements are computed applying a generalized geometry: a frustum as the volcano, an underlying halfspace and a layer of sea water surrounding the volcano.

Keywords: magnetotellurics, volcano, numerical simulation

INTRODUCTION

Stromboli volcano is 926 m high and extends down beneath the sea level to a depth of 2000 m. The first activities of the Palaeostromboli took place in the younger Pleistocene about 40,000 years ago. The characteristic Strombolian eruptions have proceeded in approximately the same manner for at least the last two thousand years.

To get a better understanding of the processes that lead to eruptions further investigations of the inner structure of the volcano are essential. As shown by Müller and Haack (2004) the magnetotelluric (MT) method might be applicable to this task. In order to calculate highly accurate results, a detailed description of the geometry of Stromboli volcano and the surrounding bathymetry is necessary. We apply 2D and 3D finite element techniques on unstructured triangular and tetrahedral to incorporate arbitrary surface and seafloor topography (Franke, Börner, & Spitzer, 2007).

First numerical simulations of MT measurements are computed applying a generalized geometry: a frustum of 3000 m height as the volcano, an underlying halfspace with a thickness of 5000 m and a layer of sea water surrounding the volcano. The electromagnetic fields, apparent resistivities and phases are calculated numerically at the seafloor, the slopes, and on top of the volcano. To resolve the upper structure of the volcano including the chimney as well as the layers underneath the volcano and the magma chamber, the computations are carried out for a wide frequency range.

Figure 1: Stromboli (picture from "strombolionline").

MAGNETOTELLURIC METHOD

The behaviour of electromagnetic fields is governed by Maxwell's equations. Assuming a harmonic time dependency $e^{i\omega t}$ as well as the magnetic and electric fields $\mathbf{H}$ and $\mathbf{E}$ as

$$\mathbf{H} = \mu^{-1}(\nabla \times \mathbf{A}) \quad \text{and} \quad \mathbf{E} = -i\omega\mathbf{A},$$

the equation of induction for the magnetic vector potential $\mathbf{A}$ reads

$$\nabla \times \mu^{-1}(\nabla \times \mathbf{A}) + (i\omega\sigma - \omega^2\varepsilon)\mathbf{A} \quad = \quad 0.$$

To solve the boundary value problem in the bounded domain Ω, electric and magnetic insulation are required for boundaries parallel ($\Gamma_{||}$) and perpendicular ($\Gamma_{\perp}$) to the current flow, respectively:

$$\begin{aligned} \mathbf{n} \times \mathbf{H} &= 0 \quad \text{on} \quad \Gamma_{||} \\ \mathbf{n} \times \mathbf{A} &= 0 \quad \text{on} \quad \Gamma_{\perp}. \end{aligned}$$

Furthermore, the magnetic field values for the top and bottom boundaries are calculated analytically for a 1D layered halfspace:

$$\begin{aligned} H_{\perp} &= 1\,\mathrm{Am}^{-1} \quad \text{on} \quad \Gamma_{top} \\ H_{\perp} &= H(z), \quad \text{on} \quad \Gamma_{bottom}. \end{aligned}$$

The conditions of continuity for the magnetic fields are valid at all interior boundaries representing possible jumps in the conductivity:

$$\mathbf{n}_1 \times \mathbf{H}_1 + \mathbf{n}_2 \times \mathbf{H}_2 = 0 \quad \text{on} \quad \Gamma_{int}.$$

To interpret MT measurements, apparent resistivity and phase are computed from the electromagnetic fields

$$\rho_{ij} = \frac{1}{\omega\mu} \mid Z_{ij} \mid^2 \quad , \qquad \phi_{ij} = arg(Z_{ij}),$$
$$Z_{ij} = \frac{E_i}{H_j} \quad , \qquad i,j = x,y$$

where Z_{ij} is the impedance for different directions of polarisation.

THE MODEL

The geometry of Stromboli volcano and the surrounding bathymetry have to be considered as detailed as possible to obtain results that are close to reality. First 2D simulations of MT measurements imply a generalized geometry depicted in fig. 2 with the following parameters: a frustum of 3 km height as the volcano, an underlying halfspace with a thickness of 100 km, a layer of sea water and an air layer of 100 km. The electrical conductivities are assigned according to Friedel and Jacobs (1997) and UNESCO (1983). Preliminary 3D calculations use an axially symmetric model (cf. fig. 3) with a 50×50×17 km sized rectangular prism surrounding the volcano.

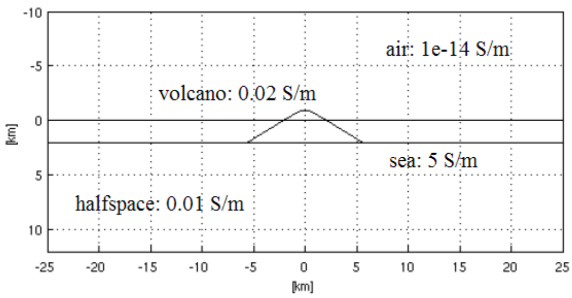

Figure 2: Section of the 2D model including the electrical conductivity distribution.

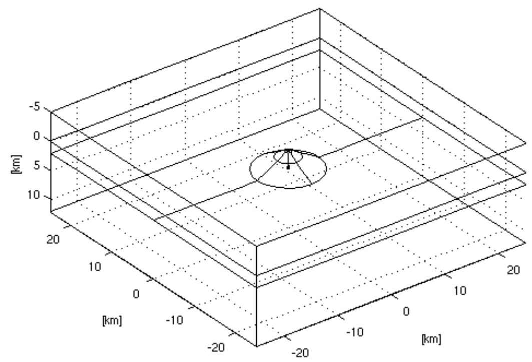

Figure 3: 3D model.

To the 2D simulations, we apply the finite element method using unstructured triangular grids and quadratic Lagrange elements. In the 3D case, tetrahedral grids and quadratic Nèdèlec elements are employed to compute the electromagnetic fields. These approaches are very well suited to take into account the steep topography and bathymetry.

MODEL STUDIES

For the first analysis of the behaviour of MT data we have carried out 2D computations. Figs. 4 and 5 display sounding curves of the apparent resistivity on the seafloor at $x = 50\,\mathrm{km}$ and on top of the volcano at $x = 0$, respectively. On the seafloor, the effect of the volcano i.e. the deviation from the halfspace resistivity of $100\,\Omega\,\mathrm{m}$ is small and limited to the period range of $10^2 ... 10^4$ s (cf.

fig. 4). These periods yield a skin depth that is larger than the thickness of the sea layer and they are suited to register a lateral effect of the resistive volcano. On top of the volcano, however, as shown in fig. 5 the apparent resistivity shows variations for periods between 10^{-2} and 10^4 s due to the conductive sea water and the underlying halfspace. Hence, the challenge is to simulate the electromagnetic fields for a wide frequency range that is suited to yield information about the conductivity distribution of the halfspace and the volcano itself.

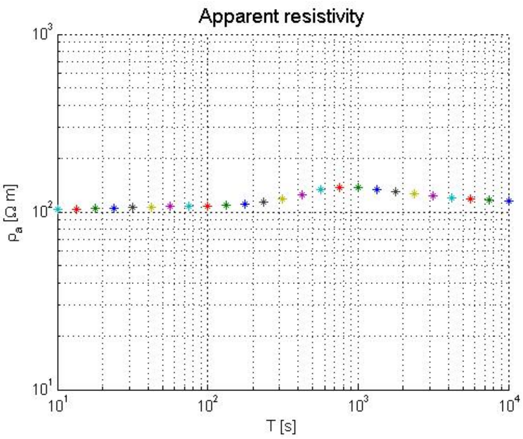

Figure 4: 2D sounding curve for E-polarisation on the seafloor at x=50km.

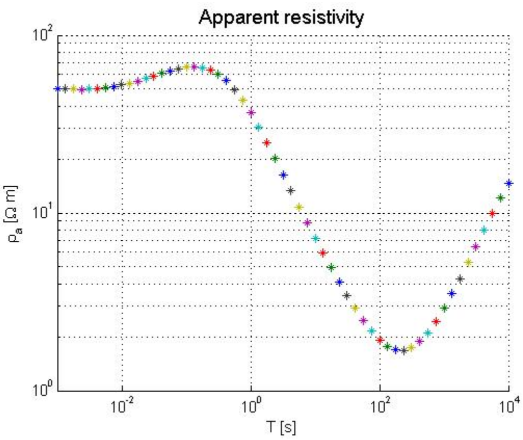

Figure 5: 2D sounding curve for E-polarisation on top of the volcano at x=0km.

The 3D simulations show a very complex behaviour of the apparent resistivity and the phase that are displayed in figs 6 and 7 for the frequency of 10^{-3} Hz on a profile. Towards the model boundaries the apparent resistivity and the phase reflect the value of the halfspace. The response associated with the volcano, i.e. the central part of the curves, is complicated due to the concurrence of topography and conductive sea water. Due to the axial symmetry of the model the calculations for xy- and yx-polarisation show the same results which confirms the correctness of the 3D computations. The calculation of accurate 3D sounding curves, however, is very memory and time consuming.

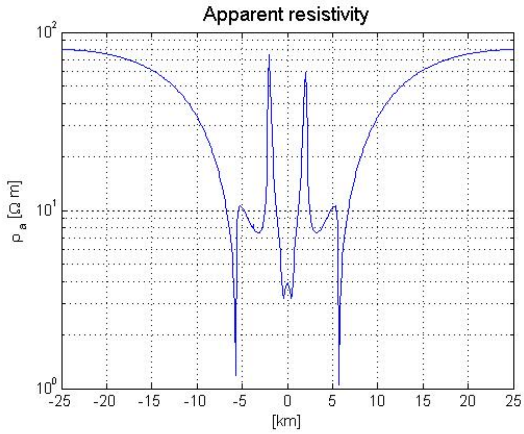

Figure 6: 3D apparent resistivity for xy-polarisation, $f = 10^{-3}$ Hz.

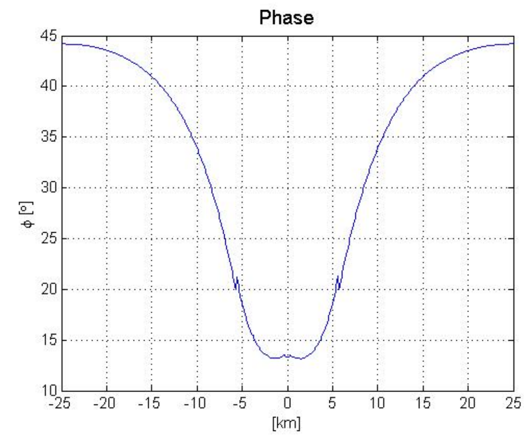

Figure 7: 3D phase for xy-polarisation, $f = 10^{-3}$ Hz.

CONCLUSIONS

We have presented our first promising results for simulating MT data at volcano Stromboli. In order to provide detailed information about the interior structure of the volcano that is of great interest with regard to eruption processes the elctromagnetic fields need to be computed for a wide frequency range and interpreted on the volcano as well as on the seafloor. We have applied the finite element method using unstructured triangular and tetrahedral grids that are well suited to take into account the topographic and bathymetric effects of the volcano's slopes. By examining the distribution of the current density and the electromagnetic fields themselves a more fundamental understanding of the underlying physical phenomena might be achieved. In the future, more detailed model studies aim at resolving rising gas bubbles associated with Strombolian eruption processes. Furthermore, to be even closer to reality we intend to use real topography and bathymetry data of Stromboli in the form of digital elevation models.

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