

Physical models for rock engineering

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1 Introduction

Physical modelling is a widespread technique to investigate the behaviour of constructions at a reduced scale. A comprehensive review about the use of scaled models in civil engineering is provided by Lirola et al. (2017). Physical modelling in geotechnical engineering is a traditional technique to investigate the behaviour of geotechnical constructions and natural geological objects (e.g. slopes, faults, caves etc.) under complex loading conditions incl. coupled hydro-thermal-mechanical-chemical ones. The loading can be highly dynamic (e.g. earthquake or impact loading) or long-term for instance creep or swelling phenomena. The main characteristic of physical modelling is that the object under investigation in nature is duplicated by a small-scale lab model with a certain scale (small scale prototype).

Compared to other investigation tools, physical modelling has the advantage, that it can duplicate very complex conditions in terms of shape, material behaviour, loading etc. Physical models also provide excellent conditions to perform measurements and monitoring and to control the test conditions (e.g. temperature, humidity, loading conditions etc.). The disadvantage is, that set-up of such models is very time consuming and costly. Also, failure states can be observed only ones, because the model will be destroyed during the experiment.

Generally, physical modelling is based on two fundamental theories:

- similarity theory
- dimensional analysis

Based on these theories, materials and corresponding properties, applied loads, the method of model construction, boundary conditions, measurements, monitoring etc. have to be chosen and adopted, respectively.

According to 1 g physical models (models under natural gravity), two approaches can be distinguished (Bakhtar, 2000):

- material scaling: geometry (size), strength and loads are scaled
- replica scaling: geometry is scaled, but strength related properties are not scaled (match original material)

Most physical modelling is performed using material scaling. A comprehensive overview about dimensional analysis and application for design of physical models in rock mechanics is given in Obert & Duval (1967).

2 Scaling

Scaling has to be done in respect to:

- model dimensions (model size)
- material properties
- loads

Physical equivalence is guaranteed by considering the similarity coefficients SC (ratio between considered quantity in-situ related to model). The geometrical similarity coefficient SC_L describes the ratio between in-situ dimension and model dimension. Exemplary, the following holds assuming an elasto-plastic Mohr-Coulomb material behaviour:

$$\frac{SC_\sigma}{SC_\gamma \cdot SC_L} = 1, \quad \frac{SC_U}{SC_\varepsilon \cdot SC_L} = 1, \quad \frac{SC_\sigma}{SC_\varepsilon \cdot SC_E} = 1, \quad \frac{SC_\sigma}{SC_C} = 1, \quad SC_\varepsilon = SC_\phi = SC_\nu = 1,$$

With:

- SC_σ similarity coefficient for stress (load),
- SC_ε similarity coefficient for deformation,
- SC_ϕ similarity coefficient for friction coefficient,
- SC_C similarity coefficient for cohesion,
- SC_E similarity coefficient for Young's modulus,
- SC_ν similarity coefficient for Poisson's ratio,
- SC_γ similarity coefficient for specific gravity,
- SC_L similarity coefficient for geometry scale,
- SC_U similarity coefficient for displacement,

If we have defined a geometrical scale SC_L and we want to adjust stiffness and strength of the materials used for the physical model, the following equations should be applied (subscript 'm' represents the model):

$$\nu_m = \nu_{in-situ} \quad \phi_m = \phi_{in-situ}$$

$$\frac{\sigma_{in-situ}^t}{\sigma_m^t} = \frac{\sigma_{in-situ}^c}{\sigma_m^c} = \frac{c_{in-situ}}{c_m} = \frac{E_{in-situ}}{E_m^t} = const. \quad \frac{F_{in-situ}}{F_m} = \frac{E_{in-situ}}{E_m^t} \cdot SC_L^2$$

where: F : applied force, E : Young's Modul, σ^t : tensile strength, σ^c : compressive strength, ν = Poisson's ratio, Φ = friction angle and c = cohesion.

The scale values for force SC_F as well as for stress and Young's modulus are:

$$SC_\sigma = SC_E = SC_L \cdot SC_\gamma \quad SC_F = SC_\gamma \cdot SC_L^3$$

If we assume a model with length scale of 1:50 and a specific gravity scale of 1:2 we obtain a force scale of 4e-6 and a stress scale of 1e-2.

Tab. 2.1: Scale factors for mechanical properties (Bakhtar, 2000)

Quantity	Dimensional form	Scale factor ⁺
Linear dimension	L	l^*
Area	L^2	l^{*2}
Volume	L^3	l^{*3}
Density	ML^{-3}	m^*l^{*-3}
Time	T	$t^{*1/2}$
Stress	$ML^{-1}MT^{-2}$	$m^*l^{*-2} = m^*l^{*-1}t^{*-2}$
Force	$MLMT^{-2}$	$m^*l^*t^{*-2} = m^*$
Velocity	LT^{-1}	$l^{*1/2}$
Acceleration	LT^{-2}	$l^*t^{*-2} = 1$
Angular velocity	T^{-1}	t^{*-1}
Mass	M	$M^* = \rho^*l^{*3}$
Energy	ML^2T^{-2}	$m^*l^{*2}t^{*-2}$
Impulse	MLT^{-1}	$m^*l^*t^{*-1}$
Strain	LL^{-1}	1
Friction angle	L^0	1
Poisson's ratio	$\Delta l_1/L_1/\Delta l_2/L_2$	1
Frequency	T^{-1}	t^{*-1}
Curvature	L^{-1}	l^{*-1}

Bakhtar (2000) has summarised some mechanical scaling factors (see Tab. 1.1) based on the basic quantities mass (m , M), length (l , L) and time (t , T). Derived quantities are then for instance: force (MLT^{-2}) or velocity (LT^{-1}). The relations become more complex in case of non-linear behaviour of the material. Bakhtar (2000) describes how dynamic processes can be scaled.

Dependent on the problem of investigation, besides the geometrical similarity, also kinematic (similarity of motion) and dynamic similarity (similarity of forces) should be considered. The general theoretical concept of physical models is based on Buckingham's π -Theorem and dimensional homogeneity of the corresponding equations (see for instance Hutter et al. (2014) or Gibbings (2011)).

Qiu et al. (2021) describe a physical model to investigate the blasting effect and give details about scaling of dynamic parameters. Li et al. (2020) describe scaling relations for hydro-mechanical coupled physical models.

3 Measurements

In principle all usual lab-based measurement techniques can be used to monitor the behaviour of a physical model. Therefore of course, the measurement scheme strongly depends on the problem under investigation. Nevertheless, a few methods are particularly important (see also Figures 3.1 to 3.3):

- High-speed cameras
- Optical deformation measurements (e.g. digital image correlation, fibre optics)
- Observation of markers placed at specific points or selected profiles (displacements, rotations)
- Monitoring via specific locally placed sensors (e.g. mechanical pressure, water pressure, temperature, fluid flow velocity etc.)
- Continuous monitoring of any kind of applied loading



Fig. 3.1: Physical model prepared for image correlation via sprayed surface (China)



Fig. 3.2: Physical model prepared with point markers to monitor movements (China)

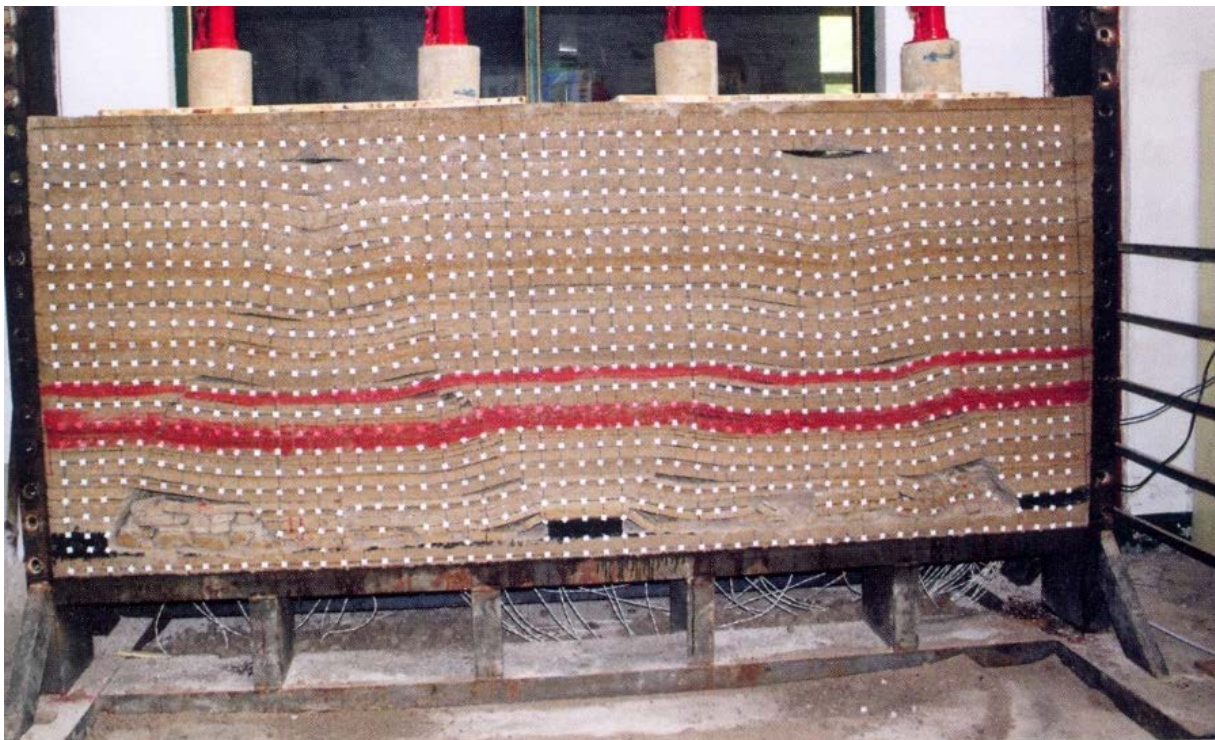


Fig. 3.3: Physical model prepared with point and profile markers to monitor movements (China)

4 Materials

The choice of materials used to construct physical models depends mainly on the specific task, model size and available loading magnitudes. Typical materials used for simulating rocks are:

- Different types of concrete
- Different types of plaster
- Materials available for 3D-printers (different plastics etc.; e.g. Song et al. 2018)
- Specific glues with different aggregates
- Special ceramics
- Weak natural rock materials
- Specific materials to represent specific construction components (e.g. special metals do represent anchors)

An overview for equivalent materials for soft rocks is given by Mei et al. (2017). Geological layering, faults, inclusions etc. can be simulated by materials with different properties like illustrated in Fig. 4.1 for a tunnel in a slightly inclined layered rock mass (sequence of coal bearing layers and sandstone / silt layers).



Fig. 4.1: Physical model with inclined layering representing strata with different properties (China)

5 Load equipment

We can distinguish mechanical, hydraulic and thermal loading. Most important is mechanical loading. Most popular are large stiff loading frames in combination with hydraulic props. They can produce uniaxial or biaxial loading under approximate plane stress conditions (see Fig. 5.1). In addition, but very rare are very large triaxial machines, which can be used also (see Fig. 5.2). Quite flexible and often used in civil engineering are movable wall and frame constructions like shown in Fig. 5.3, where hydraulic props as well as the model itself can be placed in any desired way. In case of dynamic loading so-called shaking tables (see Fig. 5.4) are used. They allow the simulation of complex earthquake loadings.

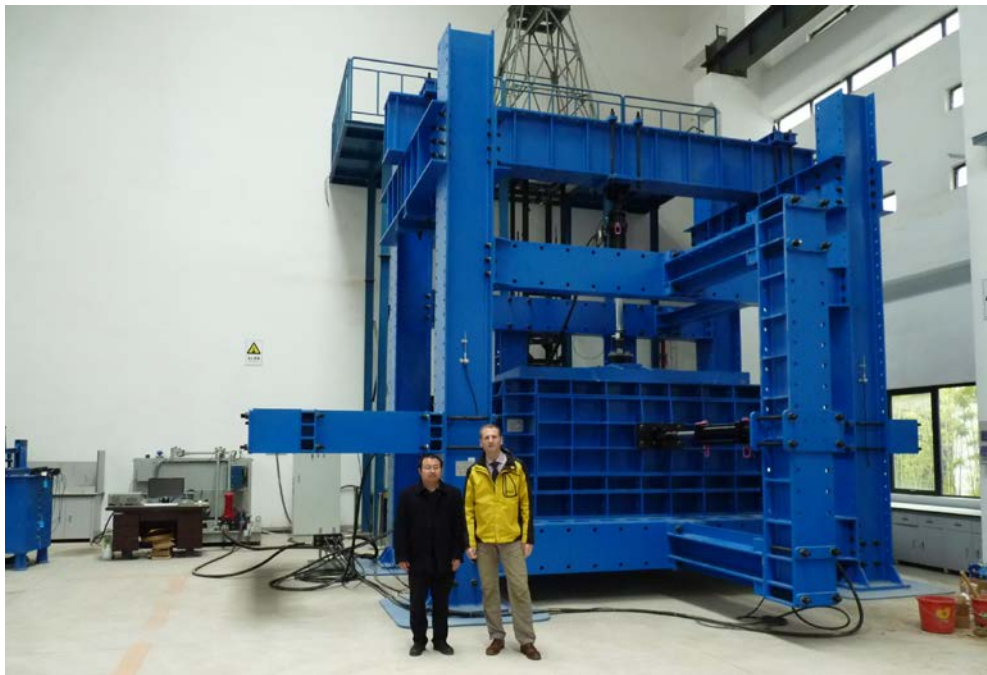


Fig. 5.1: Different loading frames (China)



Fig. 5.2: Large scale triaxial device for 3.0 m x 3.0 m x 3.5 m model / sample size (Zhengzhou, China)



Fig. 5.3: Flexible wall-frame construction usable for physical model tests (Vietnam)



Fig. 5.4: Shaking tables (left) with scaled skyscraper model (China, right)

6 Examples

6.1 Water dam

Liu et al. (2003) document a physical model test for the Three-Gorge water dam in China. Fig. 6.1.1. shows the model layout, which consists of 4 main components:

- Loading frame
- Model of dam, rock mass and power house
- Hydraulic props to apply loading
- Measurement system (LVDT's and strain gauges)

Fig. 6.1.2 shows a photo of the physical model and Fig. 6.1.3 illustrates the fracturing observed during the test applying specific loading conditions.

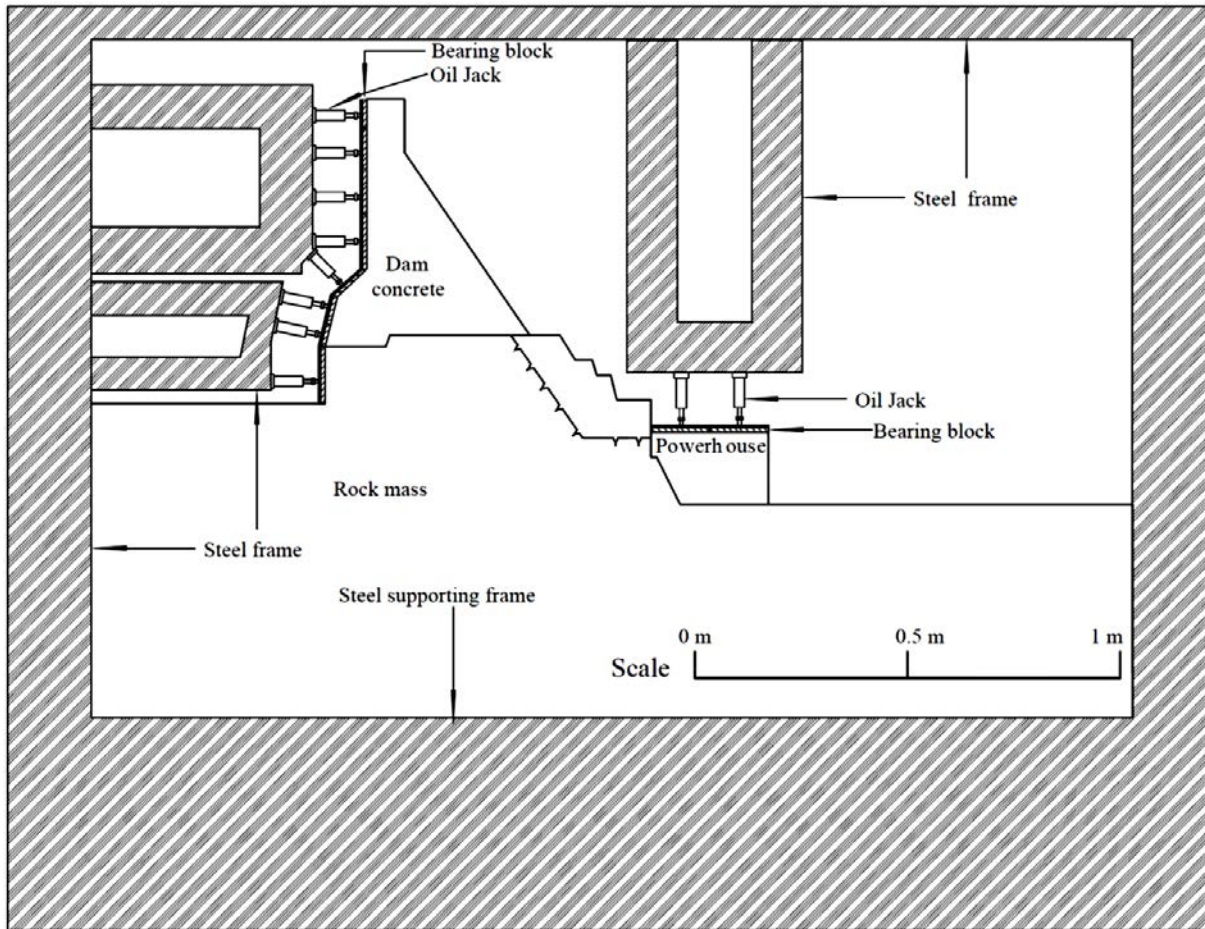


Fig. 6.1.1: Sketch of a physical dam model (Liu et al., 2003)

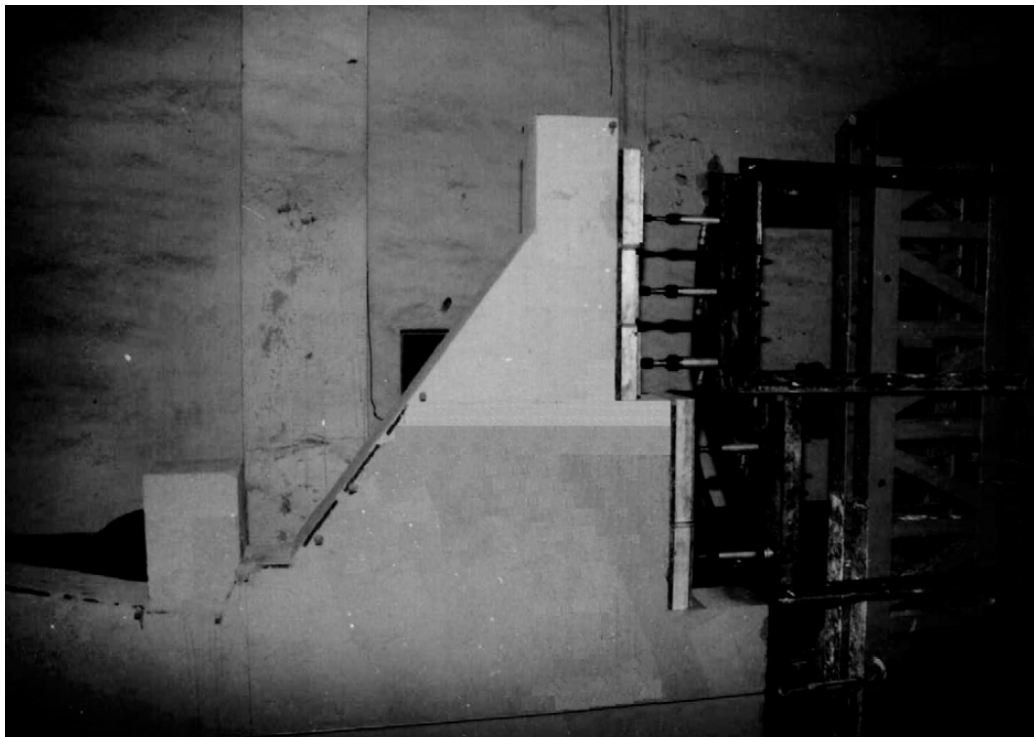


Fig. 6.1.2: Photo of a physical model (Liu et al., 2003)

Tab. 6.1.1 documents the applied scaling values, where:

- C_L : length scaling value
- C_ρ : density scaling value
- C_E : scaling for deformation modulus
- C_σ : scaling for load
- C_c : scaling for cohesion
- C_f : scaling for friction
- C_ϵ : scaling for strain

Tab. 6.1.2 shows the material parameters for the in-situ material (prototype) and the material used for the physical model.

Tab. 6.1.1: Used scaling values (Liu et al., 2003)

C_L	C_ρ	C_E	C_σ	C_c	C_f	C_ϵ
120	1.0	120	120	120	1	1

Tab. 6.1.2: Used mechanical properties (Liu et al., 2003)

Mechanical properties	Rock		Dam concrete		Joint		Foundation surface	
	Prototype	Model	Prototype	Model	Prototype	Model	Prototype	Model
Deformation modulus (GPa)	35.0	0.305	26.0	0.21	—	—	—	—
Density (kN/m ³)	27.0	26.8	24.5	24.6	—	—	—	—
Friction angle	59.6°	59.5°	48°	48°	35°	34.2°	48°	45°
Cohesion (MPa)	2.0	0.045	3.0	0.052	0.2	0.0	1.3	0.056

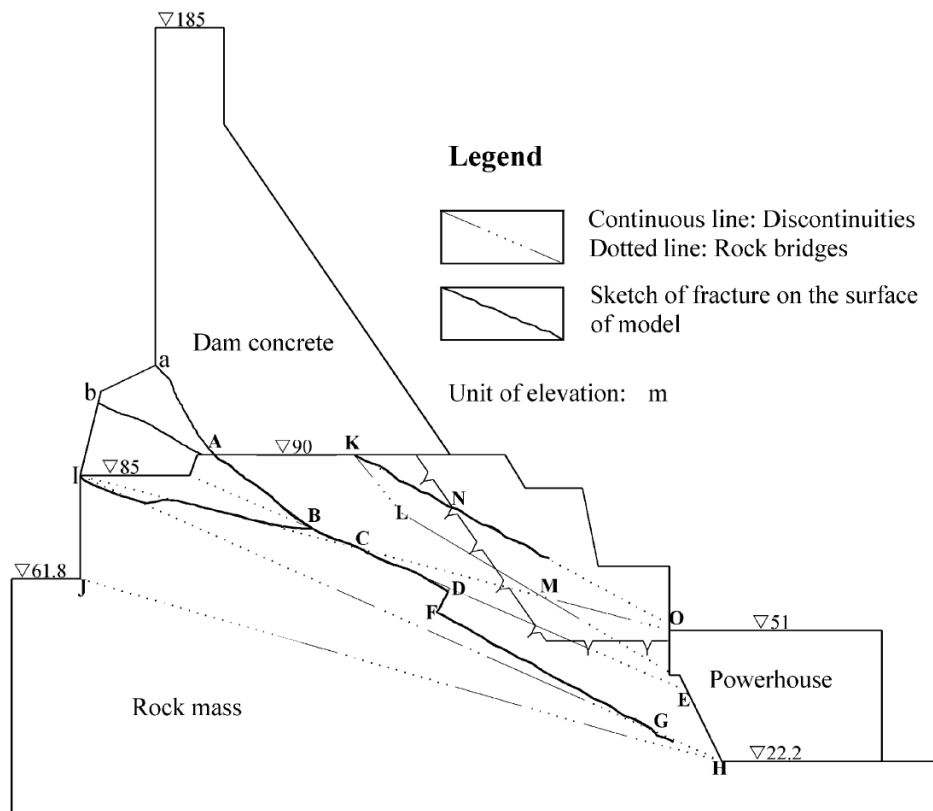


Fig. 6.1.3: Sketch of detected fractures after evaluating physical model (Liu et al., 2003)

6.2 Soil - structure interaction

Al Heib et al. (2013) describe physical model tests to investigate the soil-structure interaction due to ground movements triggered for instance by underground mining, collapse of cavities, swelling etc. The modelled foundation consists of concrete slabs and masonry structures (see Figs 6.2.1 and 6.2.2.). DIC is used as main component for monitoring. Soil is represented by special sand. The induced subsidence is controlled by a hydraulic prop (25 cm × 25 cm cross section) at the bottom centre of the model with a size of 3 m × 2 m × 1 m. Vertical movement of the prop creates a subsidence trough like observed in-situ.

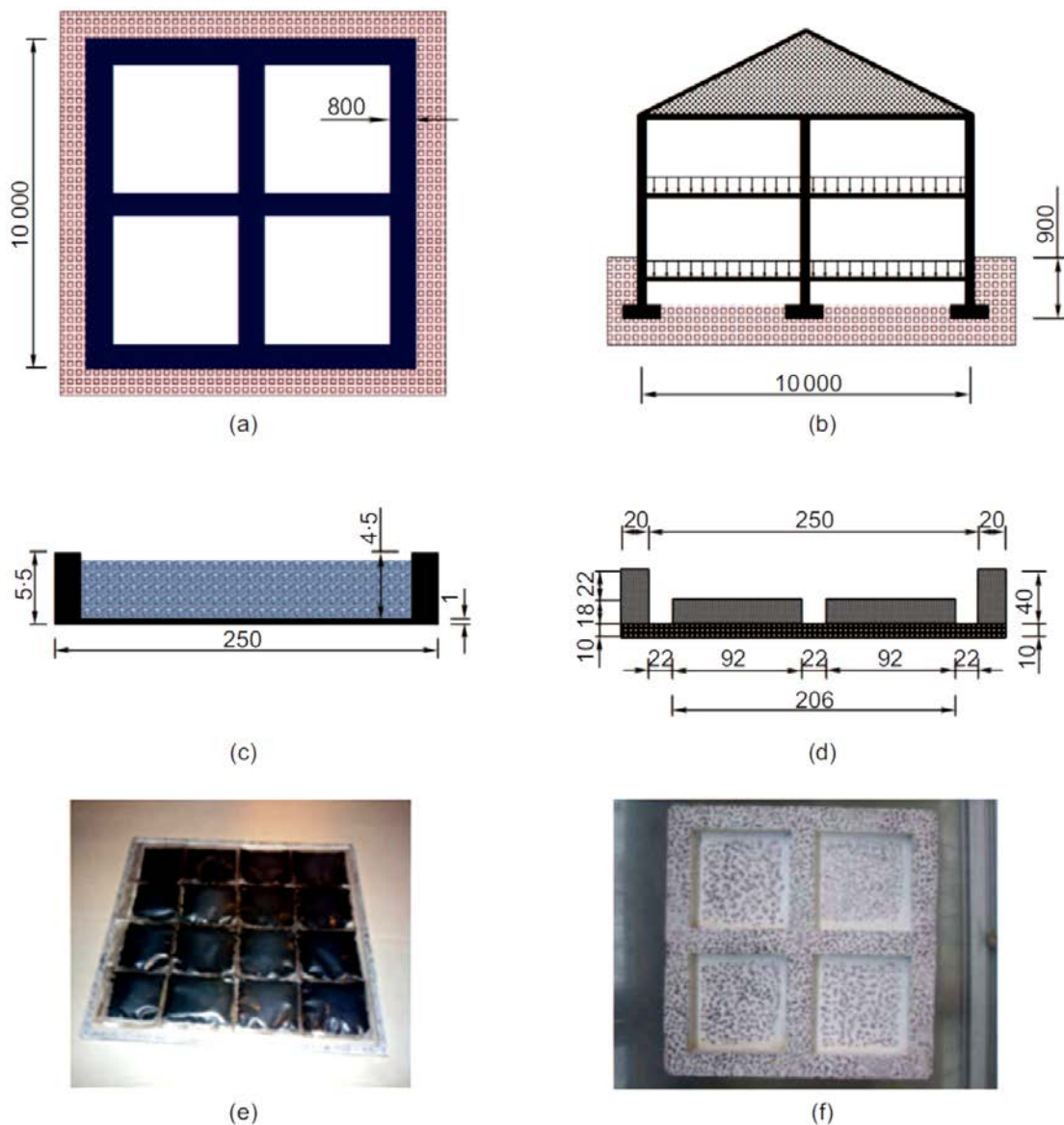


Fig. 6.2.1: Figure to illustrate set-up of physical model: a-b: in-situ situation, c-d: physical model set-up, e-f: pictures (Al Heib et al., 2013)

Tab. 6.2.1: In-situ parameters and scaled parameters for physical model according to Fig. 6.2.1 (Al Heib et al., 2013)

Parameter	Scaling factor	Prototype	Ideal model	Polycarbonate	Silicone
Width: m	40	10	0.25	0.25	0.25
Length: m	40	10	0.25	0.25	0.25
Height: mm	40	250	6.25	4.5	40
Young modulus E : MPa	40	30 000	750	2200–2500	5
Weight: kN	40^3	1000	15.6×10^{-3}	15.6×10^{-3}	21.5×10^{-3}
EA : MN	40^3	7.5×10^4	1.17	0.75	0.036
EI : N.m ²	40^5	3.9×10^4	3.81	2.86	3.3
ρ^*	1	3.9×10^{-3}	3.9×10^{-3}	3.9×10^{-3}	4.5×10^{-3}
α^*	1	2	2	2	0.096

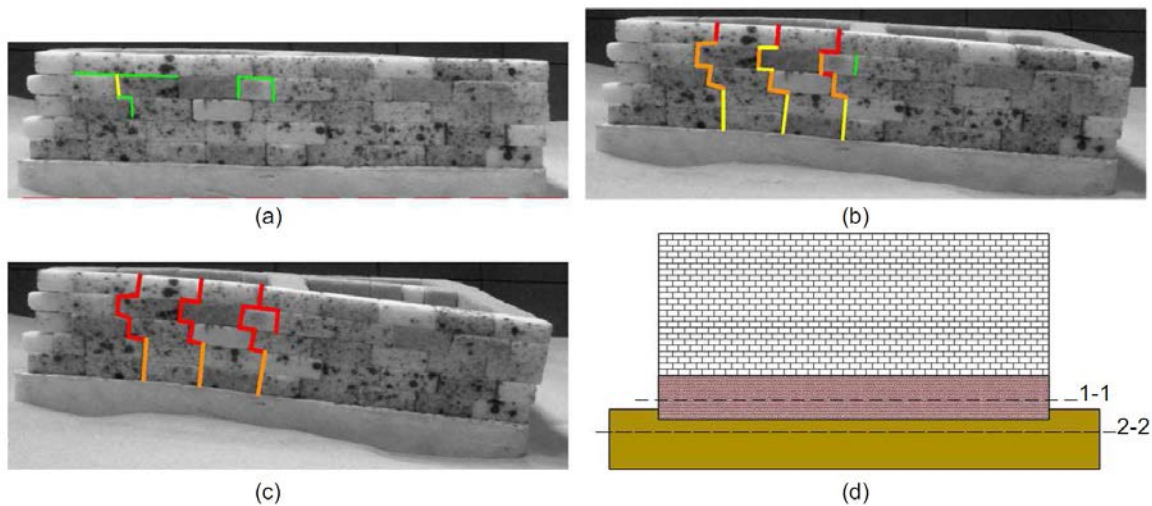


Fig. 6.2.2: Physical model of masonry walls using sugar pieces to represent bricks (Al Heib et al., 2013)

Tab. 6.2.2: In-situ parameters and scaled parameters for physical model according to Fig. 6.2.1 (Al Heib et al., 2013)

Parameter	Prototype blocks	Ideal model	Sugar	Wood
$L \cdot l \cdot h$: mm	$500 \times 250 \times 200$	$12.5 \times 6.25 \times 5$	$27 \times 18 \times 12$	$7 \times 7 \times 14$
Young modulus E : GPa	10 000		Not determined	16 000–19 000
Unit weight: kN/m ³	19.0	19.0	15.90	10.30
Friction angle between blocs φ : °	20–35	20–35	30	30 ± 9

6.3 Underground cavern system

Li et al. (2005) describe a physical model test for an underground cavern system for a hydropower station. Advanced measurement equipment is applied including micro-multi-point extensometers, AE monitoring, optical fiber sensors, internal photography and infrared micro-camera. A micro-TBM is applied to simulate the excavation advance. The chosen geometrical model scale was 1:100. Fig. 6.3.1 shows the underlying project and the corresponding physical model, which represents a part of the whole construction. Fig. 6.3.2 shows the test facility.

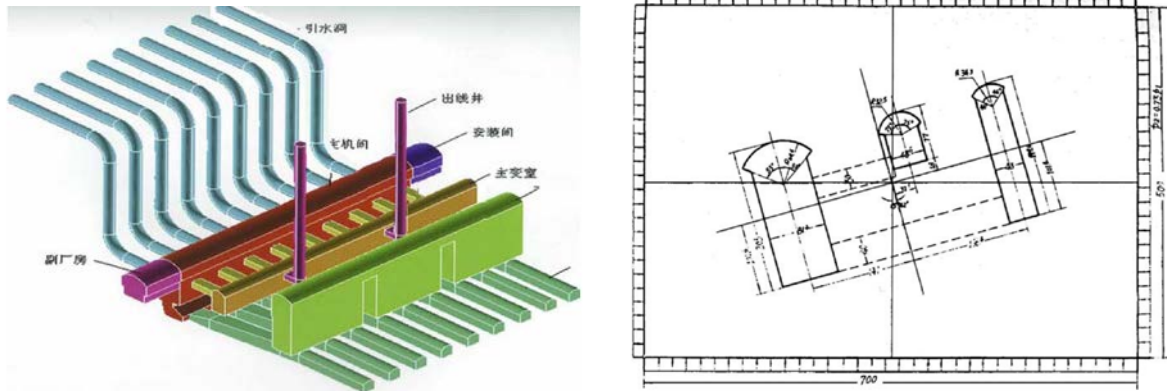


Fig. 6.3.1: Underground cavern system and layout of corresponding physical model (Li et al., 2005)

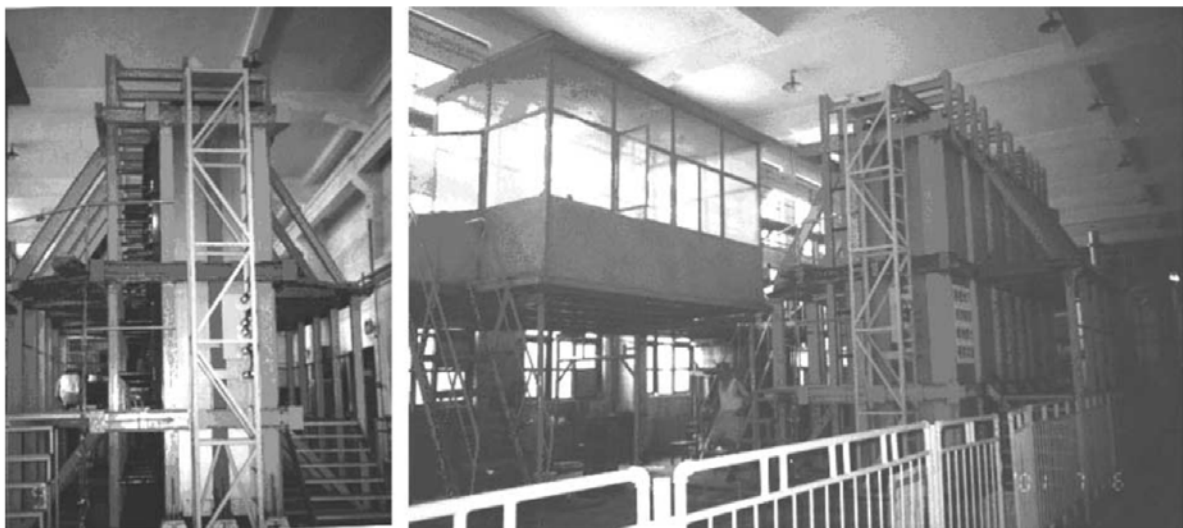


Fig. 6.3.2: Loading frame for large physical model (Li et al., 2005)

6.4 Longwall coal mining

Besides hydropower and dam projects, the physical simulation of longwall coal mining has a longstanding tradition and has reached a quite advanced level in geoen지니어ing. Already Jacobi (1976) has documented very detailed high-level physical modelling results for different elements in longwall coal mining (e.g. supported and unsupported drifts and longwall itself including overlying rock masses). The loading frame for physical longwall tests is 10 m long, 2 m high with a thickness of 0.4 m, the loading frame for physical drift tests is 2 m long, 2 m high with a thickness of 0.4 m (Fig. 6.4.1). The physical models do not only consider the behaviour of the rock mass, but contain also support elements (e.g. anchors, yielding arch support). Therefore, the scaling has to be applied also to the support elements. The typically used length scale L was 1:10. This results for instance in the following scaling relations:

- Young's modulus: 1:10
- Poisson's ratio: 1:1
- Frictional coefficient: 1:1
- Density: 1:1
- Compressive strength: 1:10
- Applied forces: 1:1000
- Moment of inertia: 1:10.000 (for structural elements)
- Cross section area: 1:100 (for structural elements)

The applied scaled support measures (see for instance Fig. 6.4.2) were well calibrated in special designed test rigs (see Fig. 6.4.3). Fig. 6.4.4. shows the damage and fracture evolution in the hanging walls as well as the development of the goaf during longwall advance. Fig. 6.4.5. illustrates the deformation and failure pattern of a drift, which is supported with yielding arch support and 2 anchors in the roof. For more recent work see also Cheng et al. (2017) or Zhou et al. (2017).

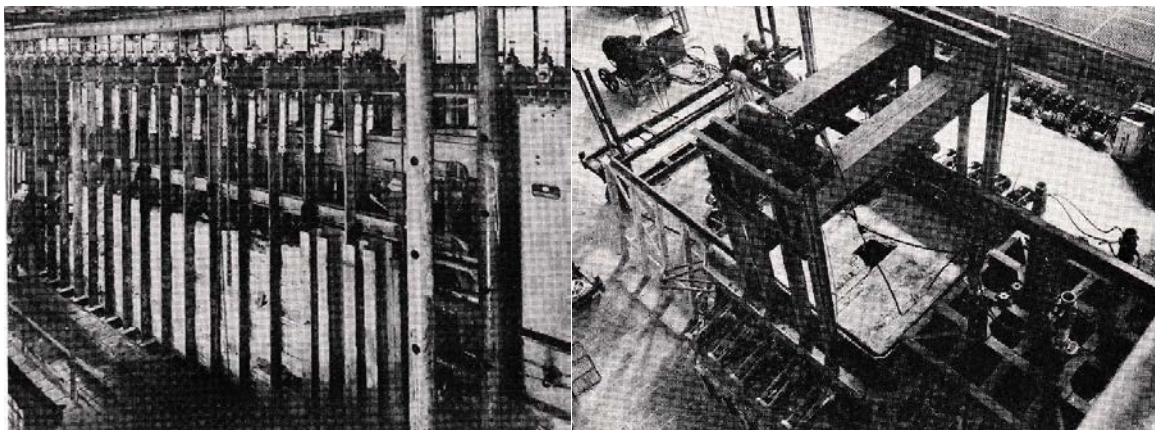


Fig. 6.4.1: Loading frames for longwall (left) and drift (right) physical models (Jacobi, 1976)

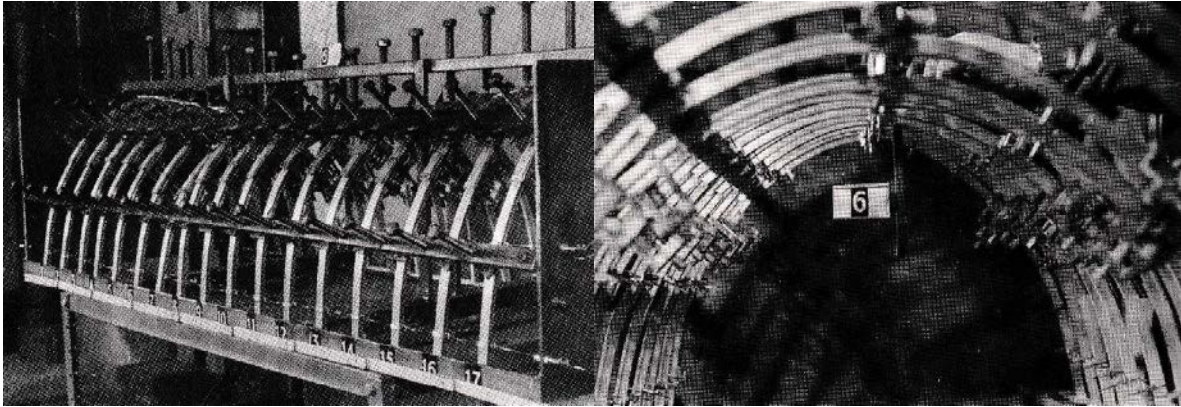


Fig. 6.4.2: Yielding arch support elements prepared for installation inside a physical drift model (Jacobi, 1976)

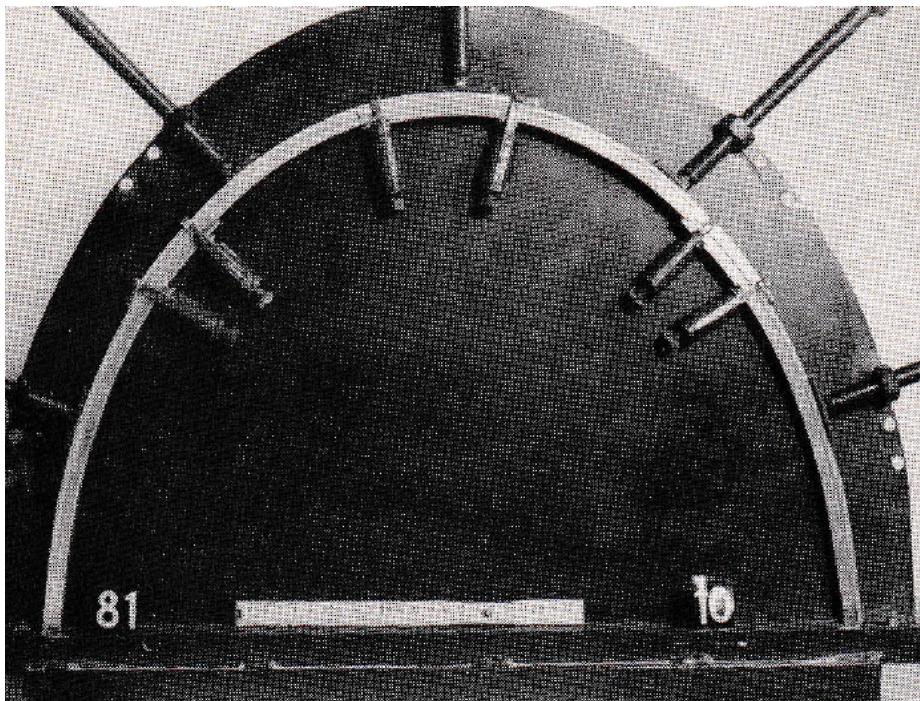


Fig. 6.4.3: Special test rig to calibrate yielding arch support elements for physical drift model with length scale of 1:10 (Jacobi, 1976)

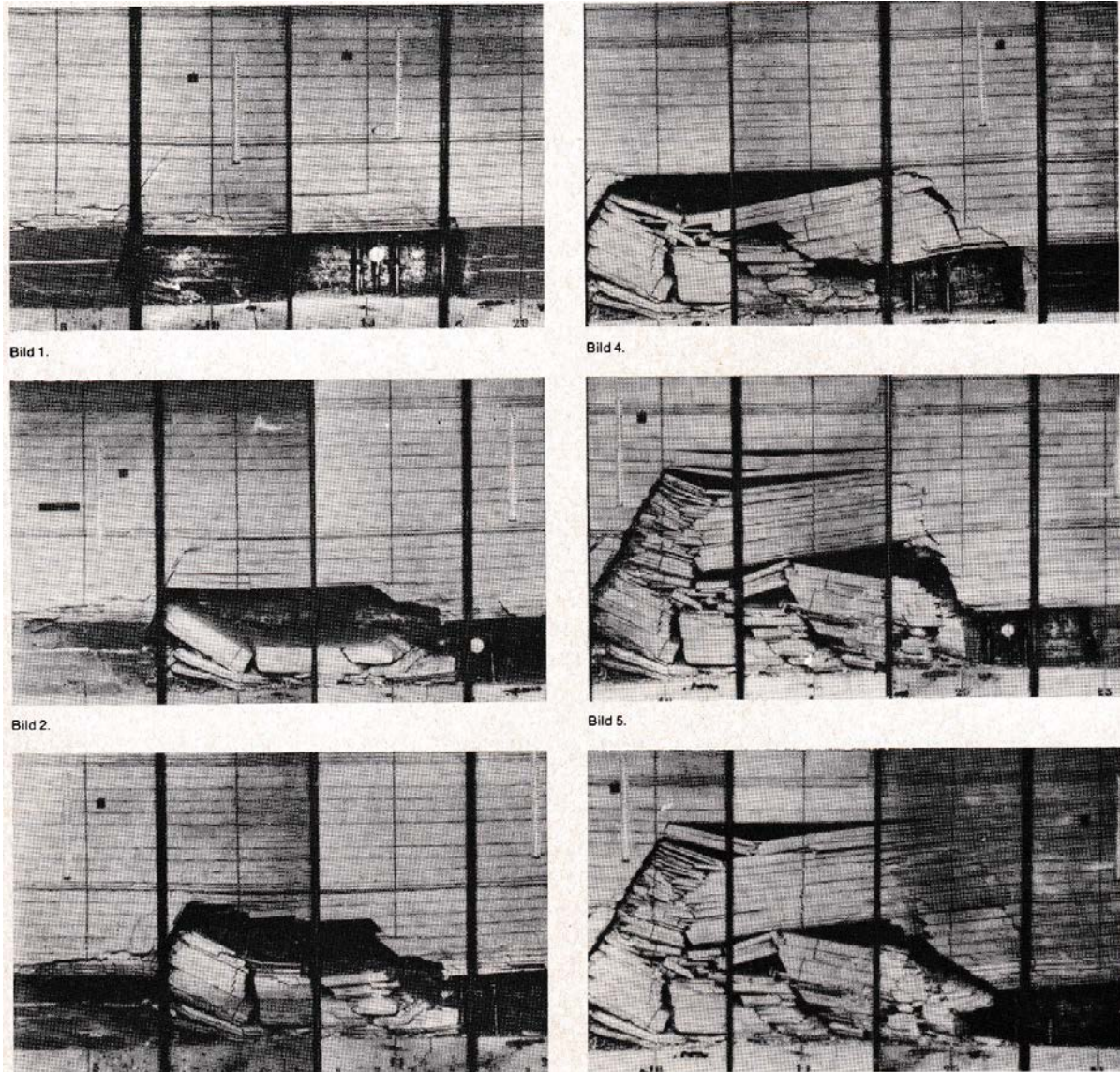


Fig. 6.4.4: Simulation of longwall advance including shield support via physical model: 6 different stages (Jacobi, 1976)

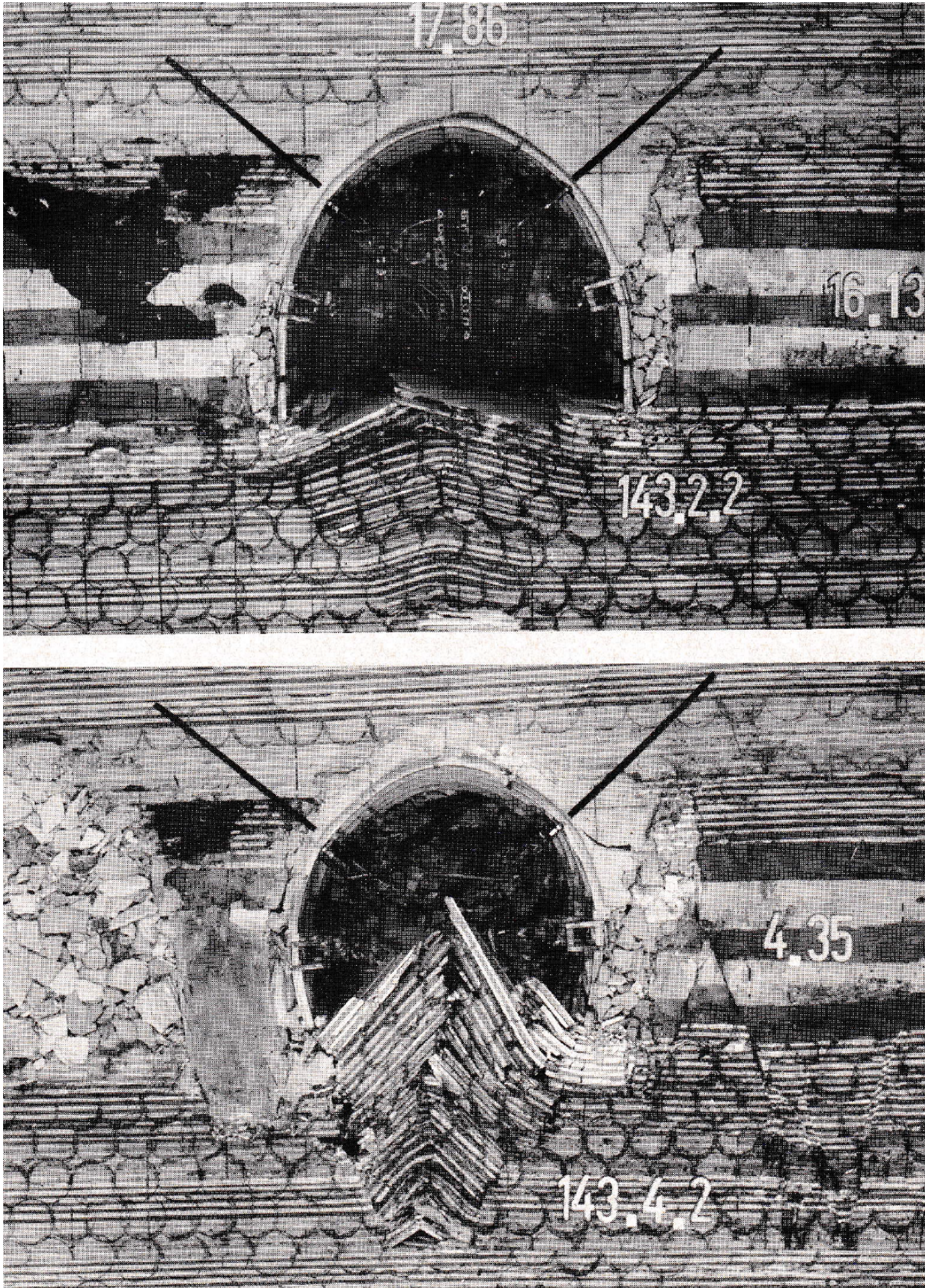


Fig. 6.4.5: Deformation and damage pattern of a supported drift with yielding support and anchors (Jacobi, 1976)

6.5 Sturzstrom simulation

Imre et al. (2010) document dynamical physical modelling using a centrifuge with the aim to investigate the Sturzstrom problem. The aim is to investigate run out, fragmentation and energy balance. The scaling has to consider particle size and size distribution, which should duplicate in-situ values, as well as scaled acceleration inside the centrifuge.



Fig. 6.5.1: Snapshot of high speed camera during test, arrow indicates movement (Imre et al., 2010)

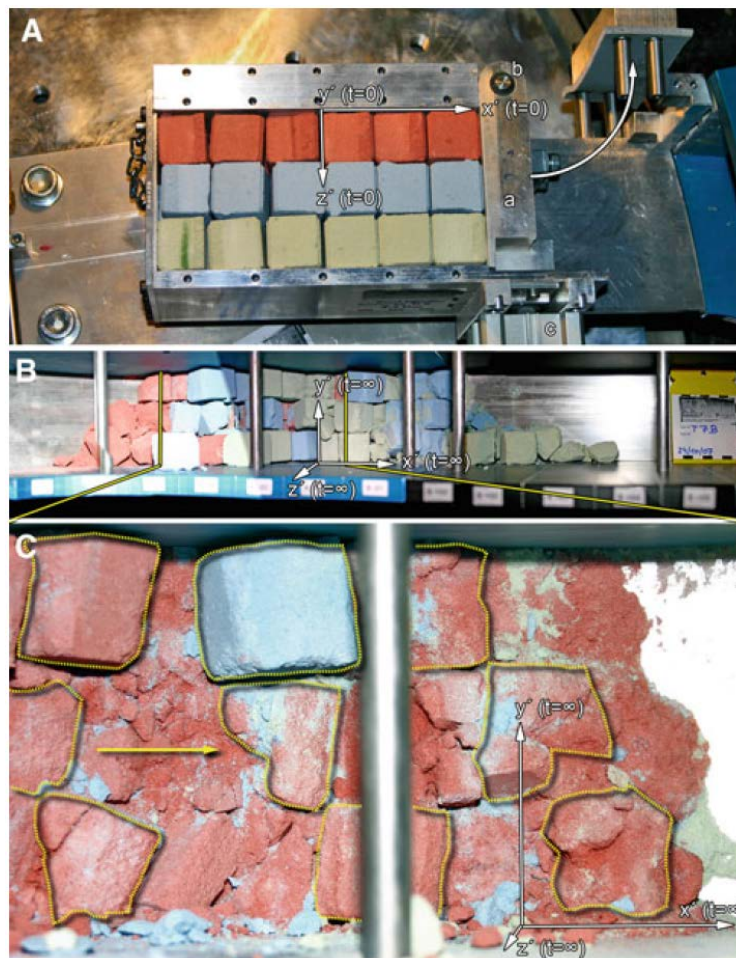


Fig. 6.5.2: Model set-up: (A) hopper section, (B) run out channel, (C) rock flow deposit (Imre et al., 2005)

6.6 Hydro-mechanical triggered slope failure

Physical models considering hydro-mechanical coupling are important for slope stability and mass flow problems. For instance, Sharma et al. (2010), Sharma & Konietzky (2011) and Jemai et al. (2017) describe slope failure model tests. High-speed cameras and sensors to measure mechanical and fluid pressure are used. The hydraulic component comprises simulation of rainfall as well as different groundwater levels (see Fig. 6.6.1).

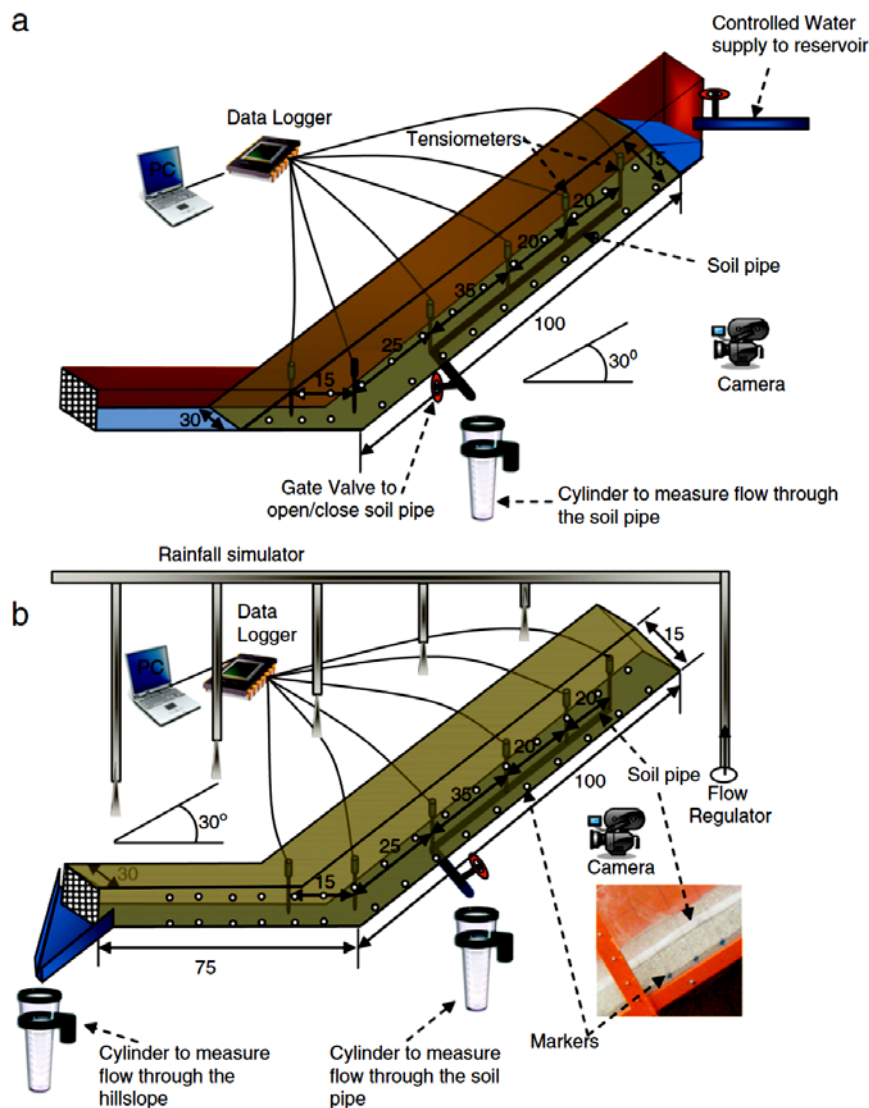


Fig. 6.6.1: Hill slope model to investigate slope failure (Sharma & Konietzky, 2011)

6.7 Rock slope failure

Slope instability due to toppling induced by an open-pit mine was investigated by Zhu et al. (2020) using a physical model with size ratio of 1:190. The chosen parameter ratios are: bulk density ratio: 1:1.5; Poisson's ratio and friction angle ratio: 1:1; stresses, deformation modulus, cohesion and total displacements: 1:285 ($= 1.5 * 190$).

Tab. 6.7.1: In-situ rock mass parameters and corresponding physical model parameters (Zhu et al. 2020)

Lithology	Uniaxial compressive strength/MPa	Tensile strength/MPa	Young's modulus/GPa	Poisson ratio	Cohesion c /MPa	Internal friction angle φ /°	Density / $\text{kg}\cdot\text{m}^{-3}$
Andalusite schist	102.6	11.9	26.9	0.17	9.8	57.4	2810
Pre-calculated model rock	0.36	0.042	0.094	0.17	0.034	57.4	1870
Prepared model rock	0.35	0.107	0.103	0.16	0.53	21.6	1920

UCS of andalusite schist is the maximum value (load normal to foliation)

Figs 6.7.1 to 6.7.3 show the general model set-up including the monitoring systems, which consist of three main components:

- Strain gauge chains (deformation measurements)
- Infrared camera (high resolution temperature measurements)
- Digital speckle displacement field measurements (DIC = Digital Image Correlation)

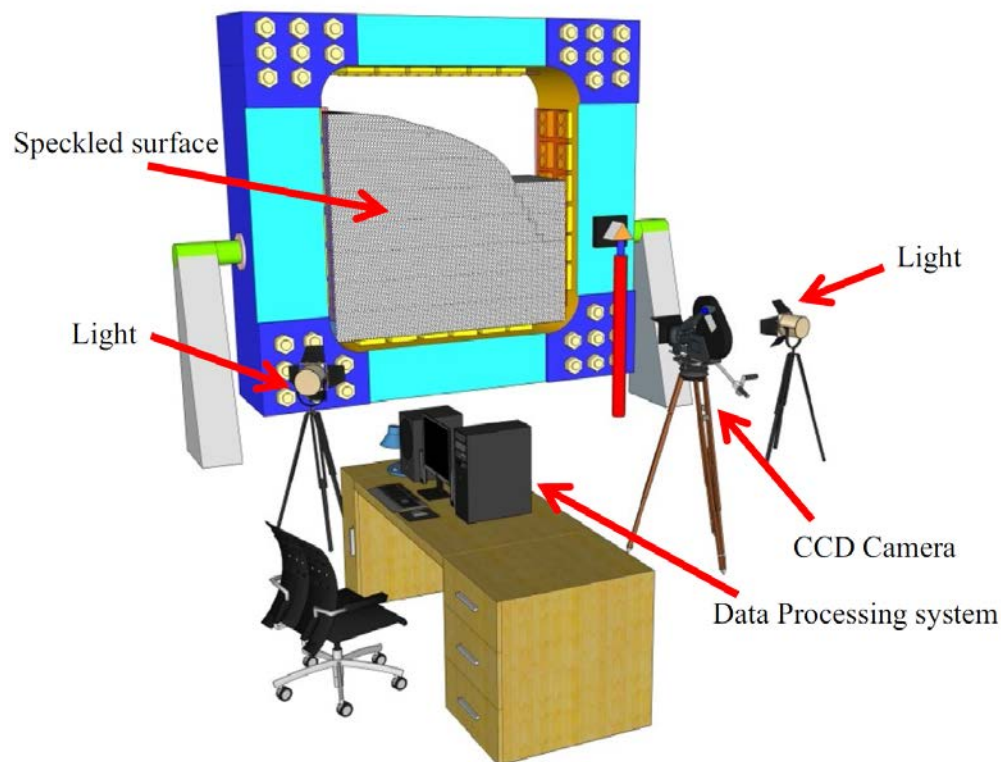


Fig. 6.7.1: Physical model set-up with DIC-system (Zhu et al. 2020)

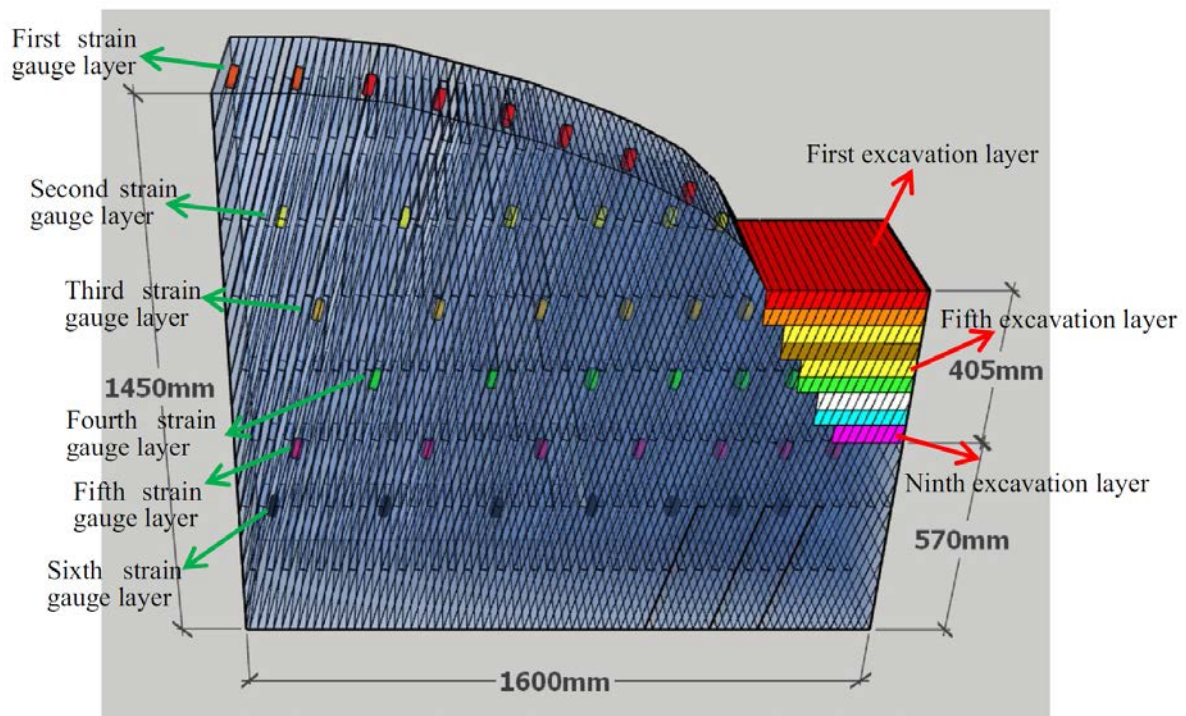


Fig. 6.7.2: Illustration of strain gauge layers in the physical model (coloured excavation layers are removed step-by-step to simulate the mining process) (Zhu et al. 2020)

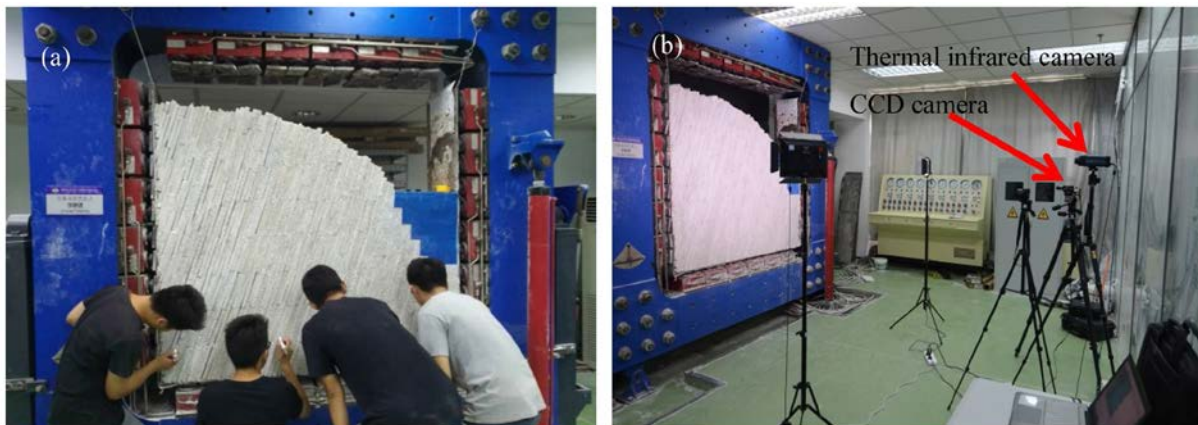


Fig. 6.7.3: Fotos of physical model incl. installed monitoring system (Zhu et al. 2020)

Figs 6.7.4 to 6.7.6 show model results in terms of failure pattern, temperature evolution and displacements triggered by simulating the open-pit mining process. Zhu et al. (2020) show, that the features observed in the physical model well duplicate the in-situ observations.

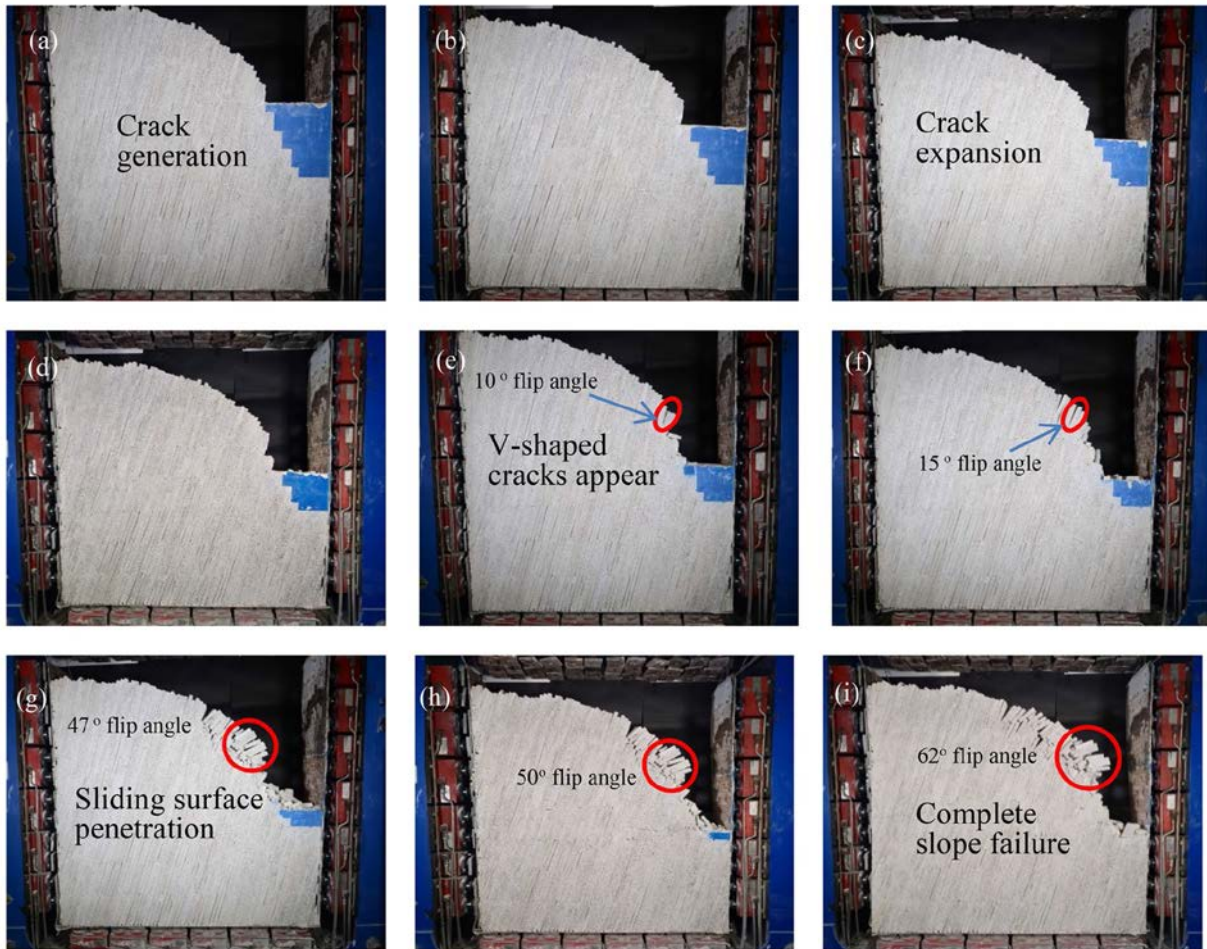


Fig. 6.7.4: Mining induced slope failure due to simulation of mining activity (stepwise removal of blue coloured layers)

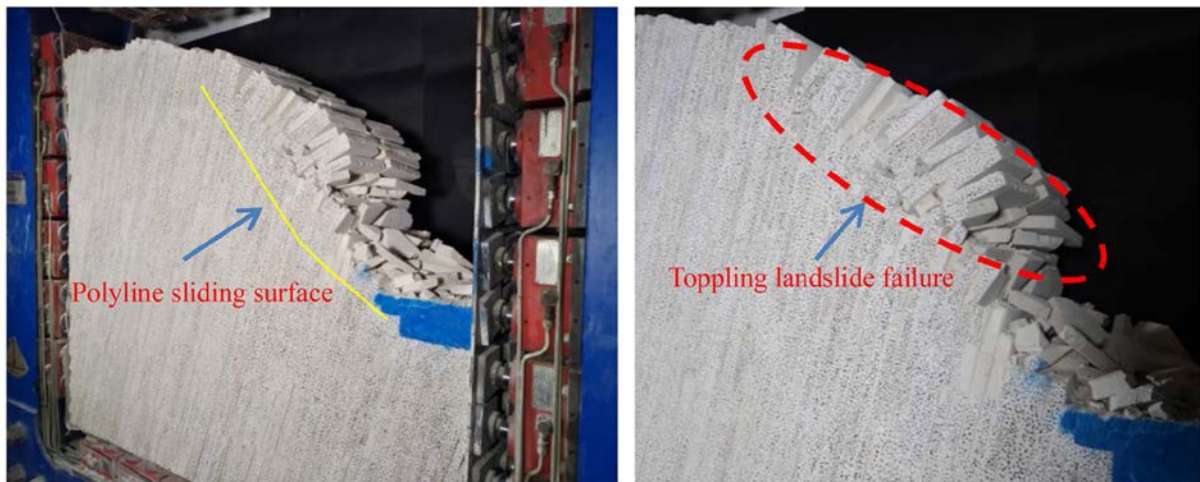


Fig. 6.7.5: Detailed view of slope failure mechanism (toppling)

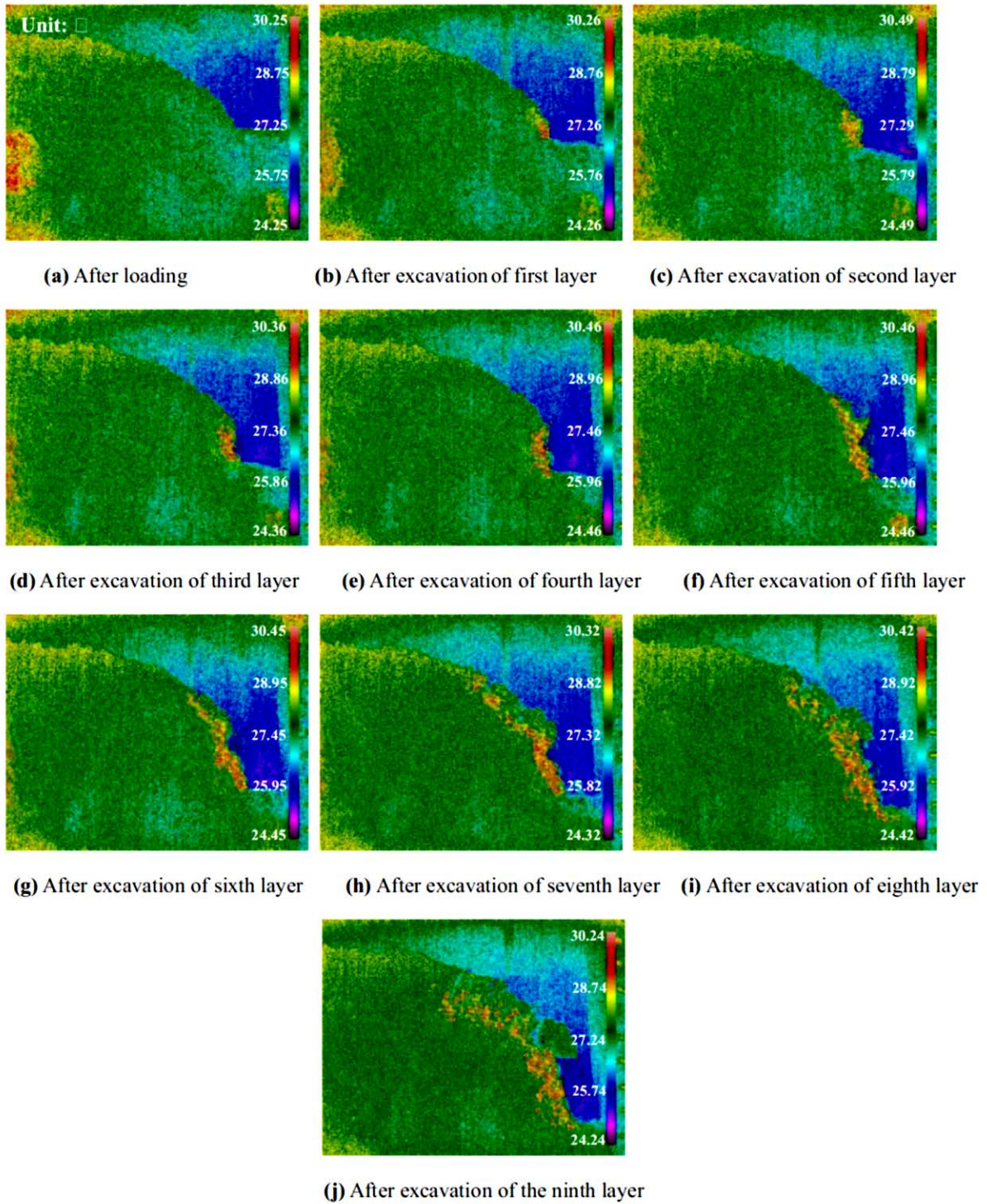


Fig. 6.7.6: Temperature evolution deduced from infrared camera observations during the simulated mining process (Zhu et al. 2020)

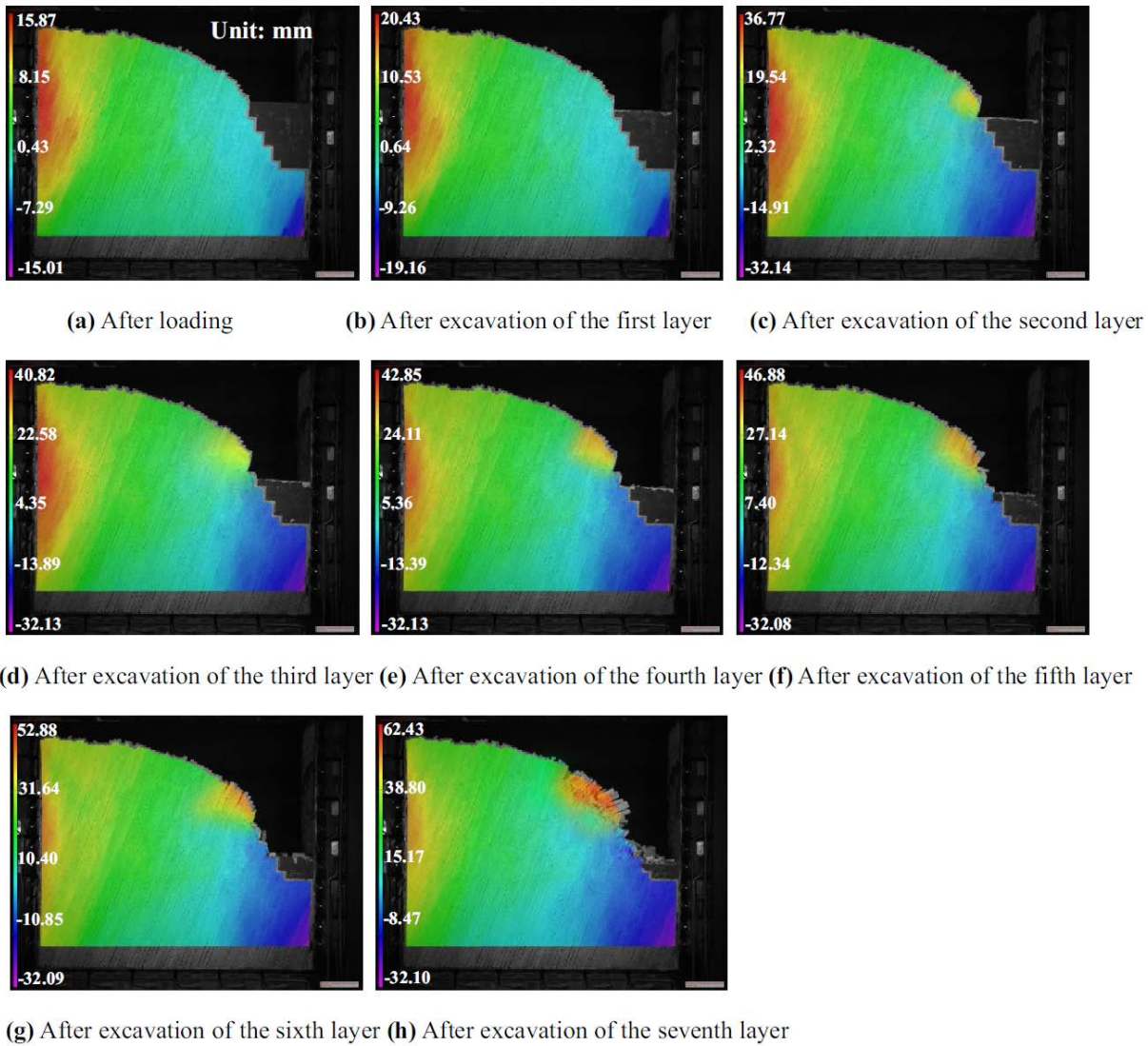


Fig. 6.7.7: Evolution of horizontal displacement component obtained from DIC measurements

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