

# Failure criteria for rocks

Authors: Prof. Dr. habil. Heinz Konietzky & M. Sc. Mohamed Ahmed Ismael  
(TU Bergakademie Freiberg, Geotechnical Institute)

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## 1 Introduction

Strength of rock masses is determined by two components: strength of rock matrix and strength of rock discontinuities (cracks, joints, fractures, pores etc. at different scales). This chapter considers only the rock matrix, which contains microcracks, flaws, pores etc. but not significant larger joints or fractures. Strength or failure of rocks (rock matrix) can be described by stress criteria, energy criteria or strain criteria. In general rock matrix strength is characterized by high compressive strength and low tensile strength. It should be noticed, that strength of solids including rocks under dynamic or cyclic loading (fatigue) conditions is characterized by different parameters and partly different relations. Dynamic rock strength is highly dependent on strain rate as documented in Fig. 1. Also, other parameters have significant influence on rock strength, like:

- micromechanical damage state (micro cracks, micro flaws, micro pores etc.)  
→ increasing number of micro cracks reduces strength (Figures 2 and 3)
- temperature  
→ increasing temperature reduces strength (Fig. 4)
- loading duration  
→ increasing loading duration reduces lifetime by increasing internal damage due to subcritical crack growth (reduction in strength) (Figures 5 and 6)
- action of fluids and chemical agents  
→ water pressure and aggressive chemical agents reduce strength

Within the last decades a huge number of failure criteria were proposed, but only a few have found wider application. The following chapters present a few selected popular failure criteria for static and quasi-static conditions used in applied rock mechanics.

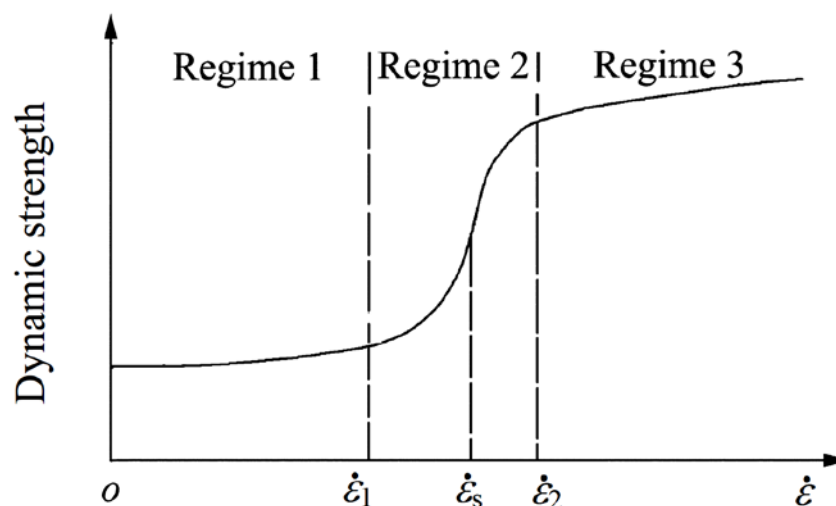


Fig. 1: Influence of strain rate on strength of rocks according to (Qian et al. 2009)  
( $\dot{\epsilon}_1 \approx 10^0 \text{ s}^{-1}$  to  $10^2 \text{ s}^{-1}$ ,  $\dot{\epsilon}_s \approx 10^3 \text{ s}^{-1}$ ,  $\dot{\epsilon}_2 \approx 10^4 \text{ s}^{-1}$ )

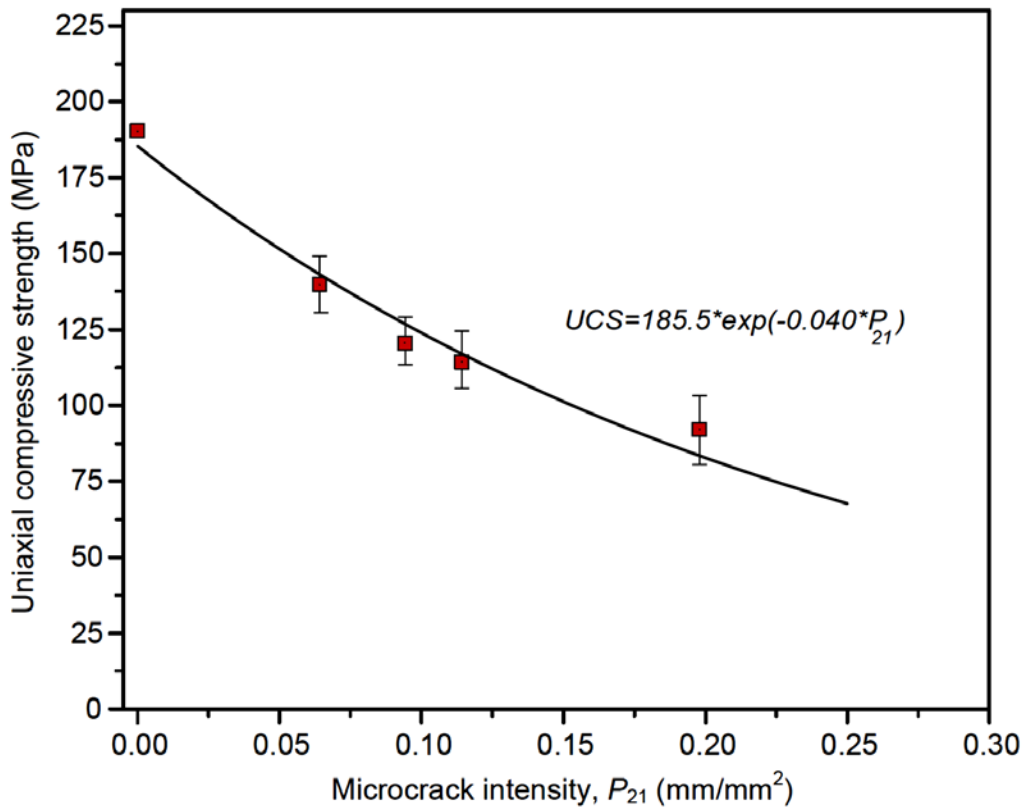


Fig. 2: Influence of micromechanical damage state on strength (Hamdi et al. 2015)

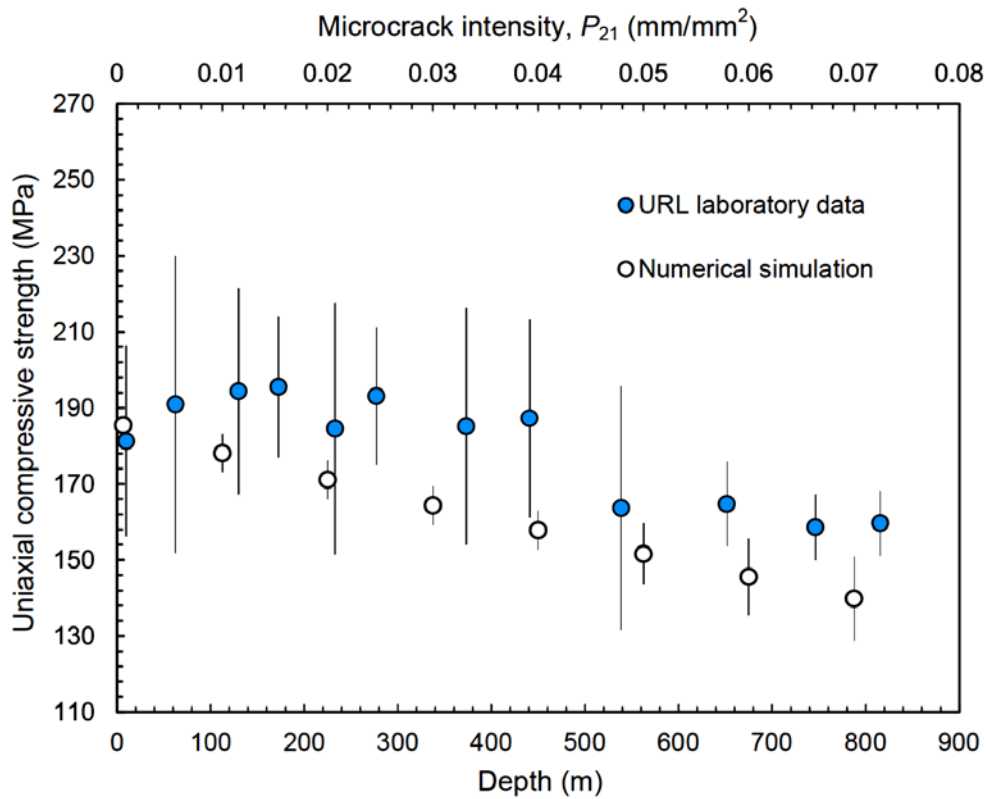


Fig. 3: Influence of depth and micromechanical damage state on strength (Hamdi et al. 2015)

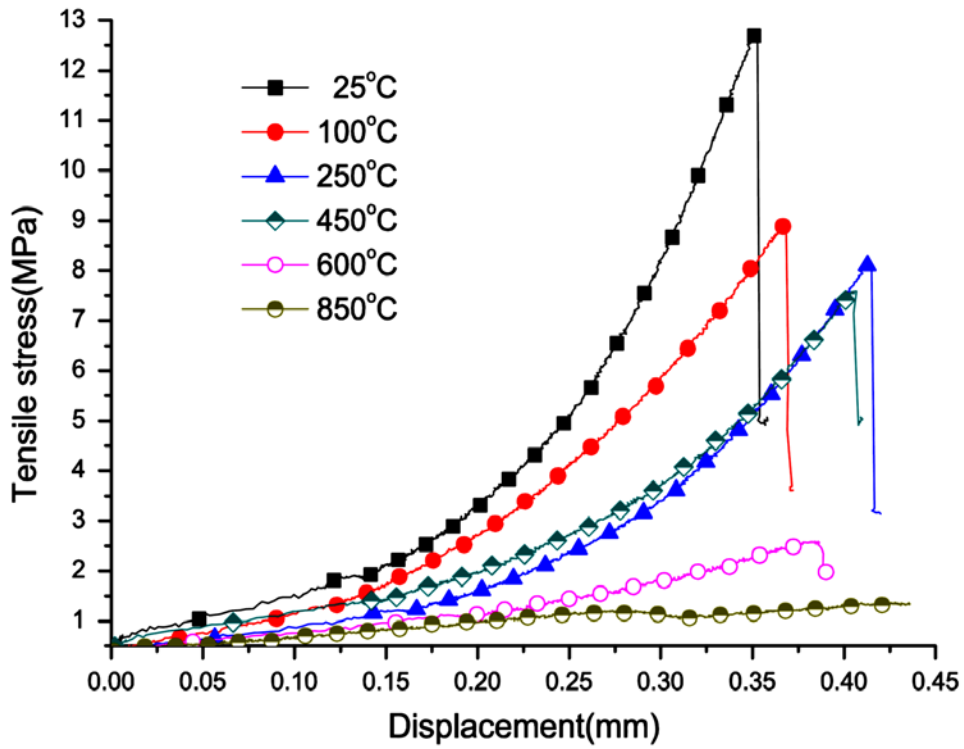


Fig. 4: Stress-strain relations and tensile strength for granite exposed to different temperatures (Yin et al. 2015)

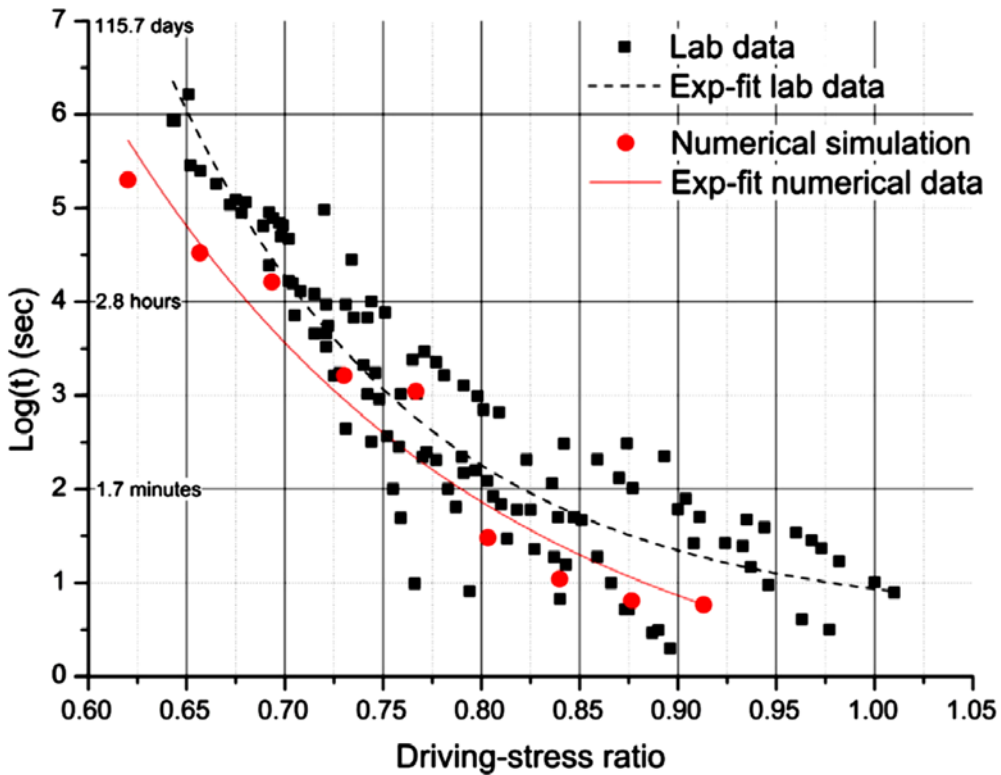


Fig. 5: Lifetime of uniaxial loaded granite samples (creep tests): lab data and numerical simulation results (Chen & Konietzky 2014)

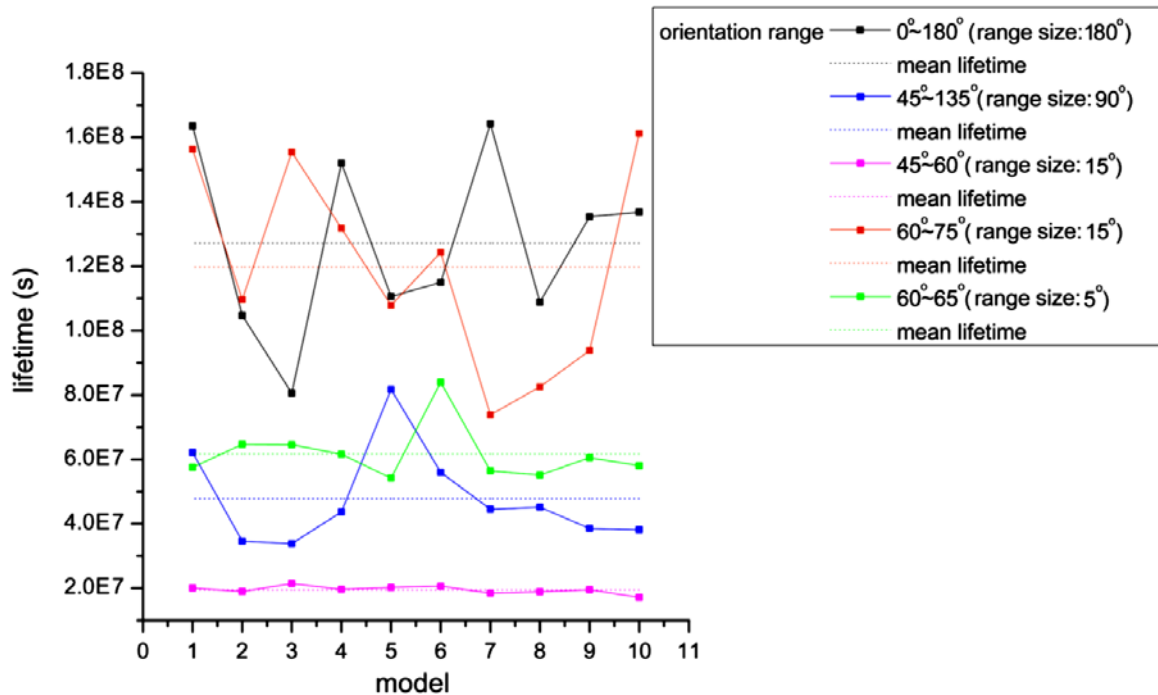


Fig. 6: Influence of orientation of microcracks on lifetime of samples under uniaxial creep loading (Li & Konietzky 2015)

## 2 Stress failure criteria

Stress criteria are the most popular type of failure criteria in rock mechanics. Most of them consider only minimum ( $\sigma_3$ ) and maximum ( $\sigma_1$ ) principal stresses, but more advanced ones include also the intermediate principal stress ( $\sigma_2$ ) component.

Fig. 7 shows a typical rock failure envelope for the 3-dimensional stress state in form of two diagrams. The colors have the following meaning:

- Red (III): triaxial tension
- Green (IV): triaxial compression
- Yellow (II): biaxial tension and uniaxial compression
- Blue (I): biaxial compression and uniaxial tension

This Fig. 7 also marks the characteristic pressure values, like:

- $\sigma_{tt}$  triaxial tensile strength
- $\sigma_{bt}$  biaxial tensile strength
- $\sigma_{ti}$  uniaxial tensile strength
- $\sigma_{ci}$  uniaxial compressive strength
- $\sigma_{bi}$  biaxial compressive strength

Note: nearly all of the common failure envelopes for rocks do not have a close failure envelop, that means the material shows under isotropic triaxial compression no failure independent on the stress level.

Fig. 8 shows in principle the failure envelopes for the general case of  $\sigma_1 \neq \sigma_2 \neq \sigma_3$ . Kwasniewski (2012) documented in detail lab results of true triaxial tests on different types of rock. Fig. 9 shows simulation results based on lab tests on coal and subsequent conducted numerical modelling in comparison with the modified Wiebols-Cook failure criterion, which also show the influence of the intermediate principal stress component.

Especially metamorphic rocks like schist, slate or gneiss and some sedimentary rocks like claystone show pronounced anisotropy in strength. Other rocks, especially igneous rocks like granite, basalt or quartzitic rocks show only low anisotropy in strength.

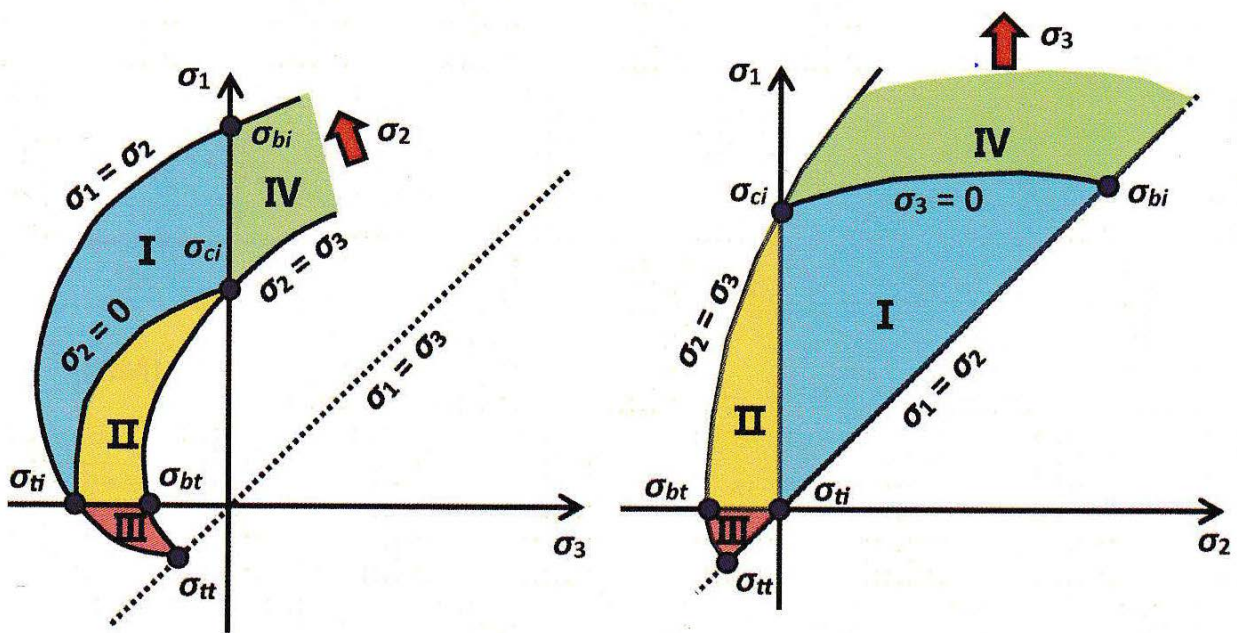


Fig. 7: Typical failure envelope for rocks under consideration of a 3-dimensional stress state (left:  $\sigma_1$ - $\sigma_3$  – diagram, right:  $\sigma_1$ - $\sigma_2$  – diagram; He et al., 2022)

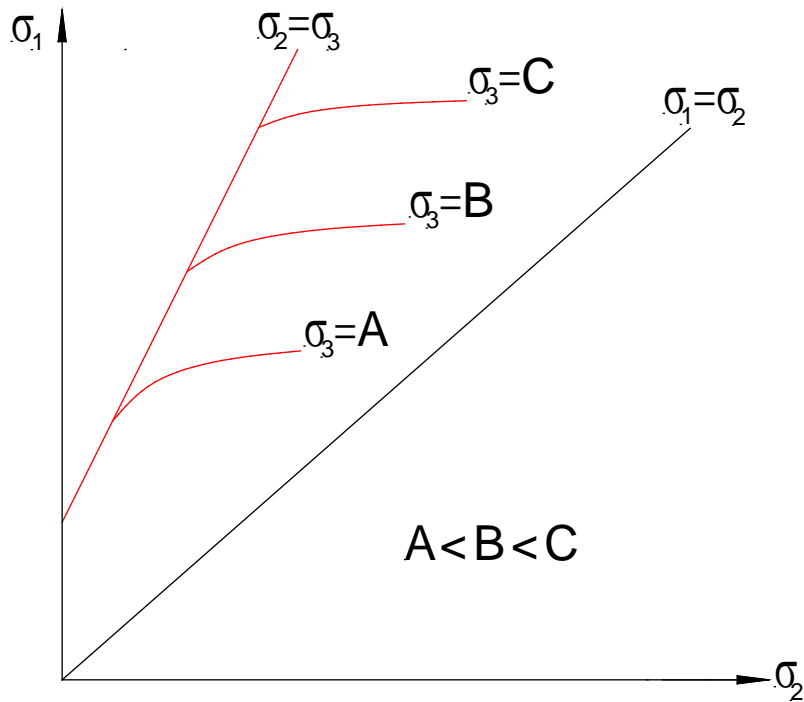


Fig. 8: General sketch for failure envelope for rocks under 3-dimensional compression for increasing confining pressure  $\sigma_3$

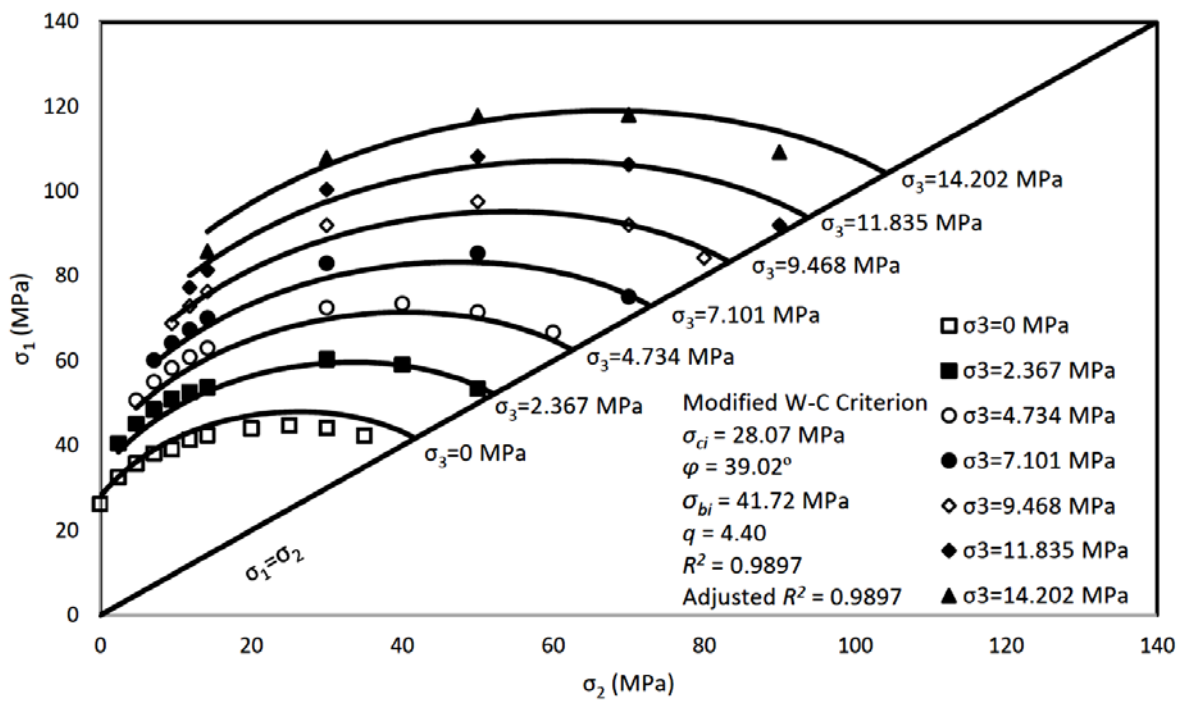


Fig. 9: Failure envelop for coal: numerical simulation results and modified Wiebols-Cook failure criterion (He et al. 2016)

## 2.1 Isotropic stress failure criteria

### 2.1.1 Mohr-Coulomb failure criterion

The classical Mohr-Coulomb failure criterion (Fig. 10) is a linear shear failure criterion and characterized by two parameters: cohesion  $c$  and friction angle  $\varphi$ . If we consider normal stress  $\sigma$  and shear stress  $\tau$ , the classical Mohr-Coulomb criterion is given as follows:

$$0 = \tau - \sigma \cdot \tan \varphi - c. \quad (1)$$

The classical Mohr-Coulomb criterion can also be given in the principal stress space, where  $\sigma_c$  is the uniaxial compressive strength:

$$0 = \sigma_1 - \kappa \sigma_3 - \sigma_c, \quad (2)$$

$$0 = \sigma_1 - \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3 - \frac{2c \cos \varphi}{1 - \sin \varphi}. \quad (3)$$

Based on equations (2) and (3) the uniaxial tensile and compressive strengths ( $\sigma_T$  and  $\sigma_c$ ) are given by:

$$\sigma_T = \frac{2c \cos \varphi}{1 + \sin \varphi} \text{ and } \sigma_c = \frac{2c \cos \varphi}{1 - \sin \varphi}. \quad (4)$$

Consequently, the ratio between uniaxial compressive and tensile strength is given by following relation:

$$\frac{\sigma_c}{\sigma_T} = \frac{1 + \sin \varphi}{1 - \sin \varphi}. \quad (5)$$

Assuming realistic values for friction angle, the resulting strength ratio according to equation (5) is quite small (factor of about 3) and is not in agreement with measurements on rocks, which show values between about 5 and 20. Therefore, for rock mechanical applications the classical Mohr-Coulomb criterion is often extended by a tension cut-off criterion  $\sigma_3 = \sigma_T$  as illustrated in Fig. 10.



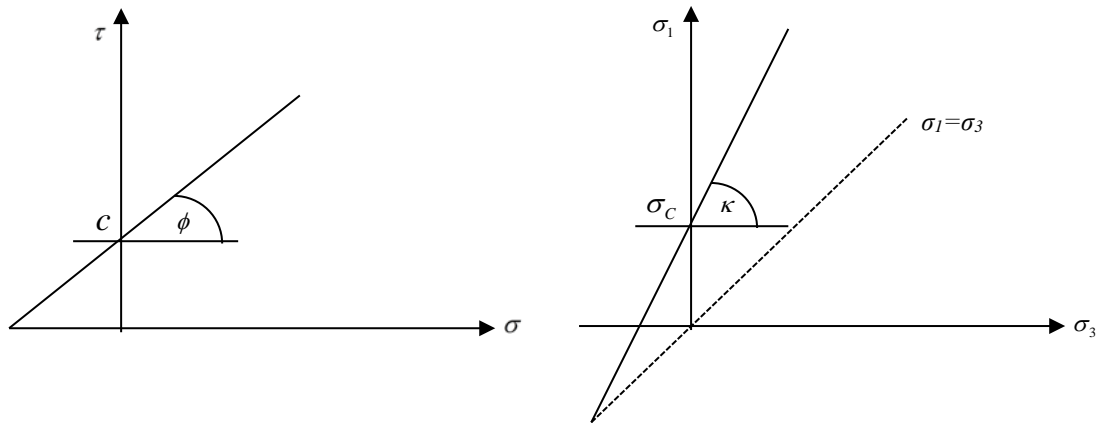


Fig. 10: Classical Mohr-Coulomb failure criterion represented in the shear-normal stress diagram (left) and in the principal stress diagram (right)

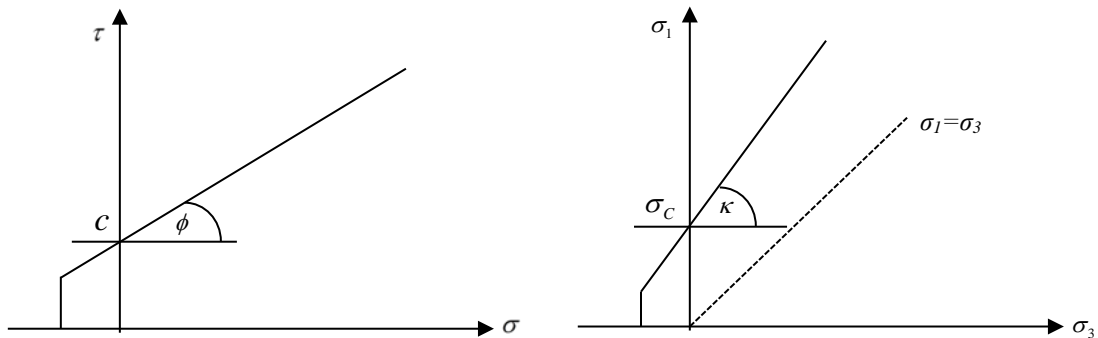


Fig. 11: Mohr-Coulomb failure criterion with tension cut-off represented in the shear-normal stress diagram (left) and in the principal stress diagram (right)

If the friction angle is set to zero the Mohr-Coulomb criterion is transferred into the so-called Tresca criterion:

$$0 = c + \sigma_3 - \sigma_1. \tag{6}$$

### 2.1.2 Drucker-Prager failure criterion

The Drucker–Prager failure criterion is illustrated in Fig. 12. It is similar to the Mohr-Coulomb criterion and is often used because it creates a cone as failure envelope in the 3-dimensional stress space instead of a six-sided pyramid in case of the Mohr-Coulomb criterion (Fig. 10).

The Drucker-Prager criterion is based on the first main stress invariant  $\sigma_0$  and the second basic deviatoric invariant  $J_2^D$  and needs to material parameters  $q$  and  $K$ :

$$0 = \sqrt{\frac{1}{2} s_{ij} s_{ij}} - q \frac{\sigma_{kk}}{3} - K. \tag{7}$$

$$0 = \sqrt{J_2^D} - q \sigma_0 - K$$

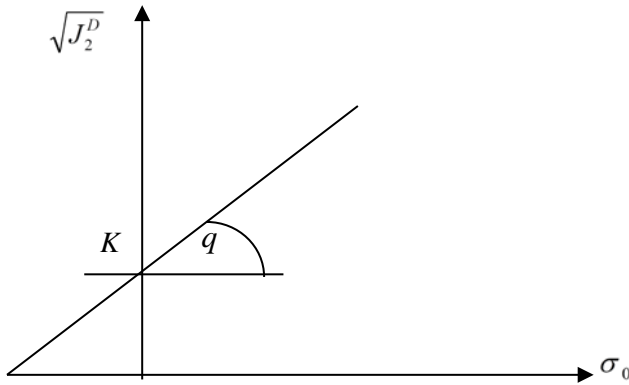


Fig.12: Drucker-Prager criterion

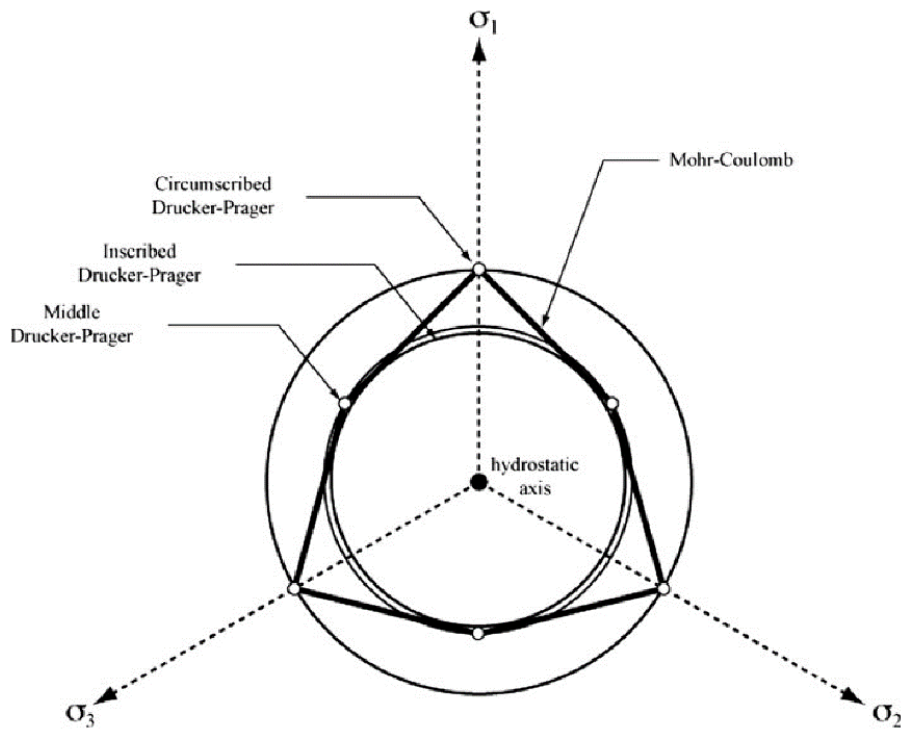


Fig. 13: Drucker-Prager and Mohr-Coulomb failure criteria in stress space (Alejano & Bobet 2012)

In relation to the Mohr-Coulomb failure criterion the parameters of the Drucker-Prager criterion can be adjusted in such a way that they inscribe or circumscribe the Mohr-Coulomb envelop. For inscribed Drucker-Prager criterion holds:

$$K = \frac{3C \cos \varphi}{2 \sqrt{\frac{1 + \sin \varphi}{1 - \sin \varphi} \sqrt{9 + 3 \sin^2 \varphi}}} \text{ and} \quad (8)$$

$$q = \frac{3 \sin \varphi}{\sqrt{9 + 3 \sin^2 \varphi}}. \quad (9)$$

For circumscribed Drucker-Prager criterion holds:

$$K = \frac{\sqrt{3}C}{2 + \frac{1 + \sin \varphi}{1 - \sin \varphi}} \text{ and} \quad (10)$$

$$q = \frac{\sqrt{3} \left( \frac{1 + \sin \varphi}{1 - \sin \varphi} \right)}{2 + \frac{1 + \sin \varphi}{1 - \sin \varphi}}. \quad (11)$$

Compared to the experimental results, the Drucker-Prager failure criterion tends to overestimate the rock strength (Alejano & Bobet 2012). Generally, this criterion is suitable for low friction angles (less than 30°) typical for sand, cement or rocks (Vermeer & De Borst 1984).

### 2.1.3 Von-Mises failure criterion

The von-Mises criterion is a very popular and simple criterion often used in material sciences, especially as reference value for graphical presentations. The von-Mises yield criterion does not depend on the mean stress. It contains only one material parameter  $K$  which represents the undrained shear strength under pure shear conditions. Also, this parameter relates the criterion to the principal stress differences:

$$0 = (\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 - 6K^2. \quad (12)$$

### 2.1.4 Hoek-Brown failure criterion

Lab test results indicate that the failure envelop is nonlinear. Hoek et al. (2002) have developed a simple empirical law based on the analysis of a huge number of lab tests and have proposed the following relation:

$$0 = \sigma_3 + \sigma_{ci} \left( m_b \frac{\sigma_3}{\sigma_{ci}} + s \right)^a - \sigma_1, \quad (13)$$

where  $m_b$ ,  $a$ ,  $s$ ,  $\sigma_{ci}$  are material parameters. Typical values for intact rock are  $a = 0.5$  and  $s = 1$ ). According to equation (13), which is the formulation of the Hoek-Brown failure criterion for intact rock,  $m_i$  is an intact rock parameter:

$$0 = \sigma_3 + \sigma_{ci} \left( m_i \frac{\sigma_3}{\sigma_{ci}} + 1 \right)^{0.5} - \sigma_1. \quad (14)$$

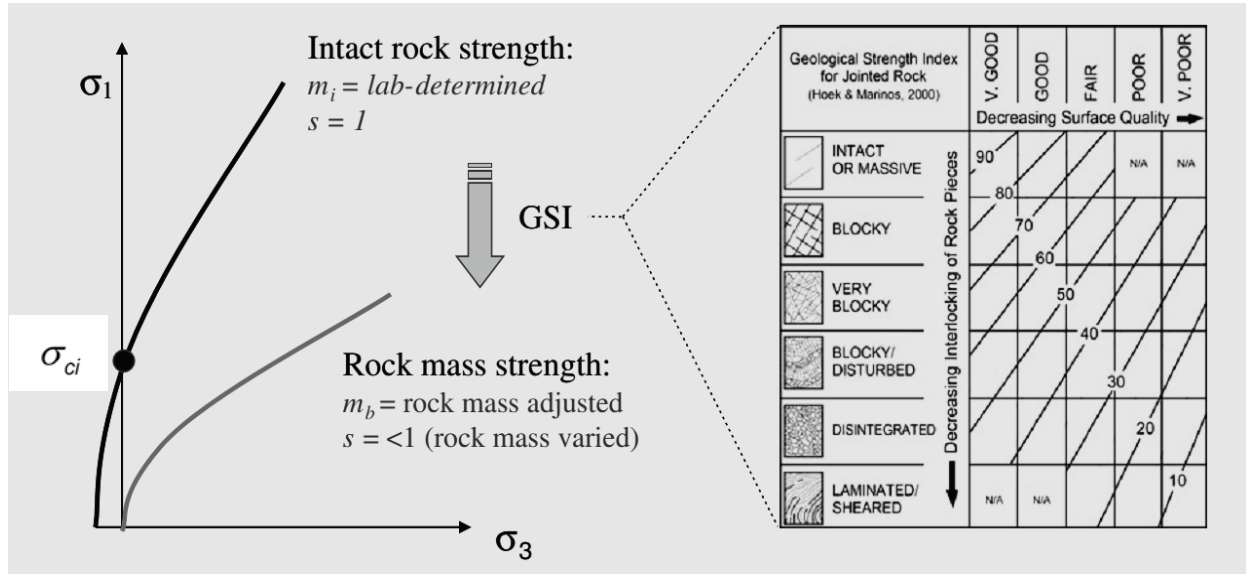


Fig. 14: Hoek-Brown failure criterion for intact rock and rock mass (Eberhardt 2012).

The intact rock parameter  $m_i$  is derived by curve fitting with experimental results obtained from triaxial tests as shown in Fig. 14. The corresponding parameter for rock mass can be derived from following relation:

$$m_b = m_i \exp\left(\frac{GSI - 100}{28 - 14D}\right), \quad (15)$$

where  $GSI$  and  $D$  are the Geological Strength Index and the Disturbance Factor respectively (Hoek et al. 2002). Fig. 14 illustrates the Hoek-Brown criterion. Due to the non-linearity the Hoek-Brown criterion is able to depict low tensile strength values and flattening of the failure envelope at large compressive stresses. For rock mass and by setting  $\sigma_3 = 0$  in equation (14), the uniaxial compressive strength is obtained by following expression:

$$\sigma_c = \sigma_{ci} s^a. \quad (16)$$

The uniaxial tensile strength is obtained by setting  $\sigma_1 = \sigma_3 = \sigma_T$  in equation (14) as:

$$\sigma_T = \frac{s\sigma_{ci}}{m_b}. \quad (17)$$

Based on the strain energy considerations, Chen et al. (2016) have deduced a similar but new strength criterion. This is a more physically based law, and it has three and in a simplified version two parameters ( $B_2 = 0$ ), which have to be obtained by curve fitting as shown in Fig. 15 ( $\nu$  is Poisson's ratio):

$$0 = 2\nu\sigma_3 + \sqrt{(4\nu^2 + 2\nu - 2 + B_2)\sigma_3^2 + B_1\sigma_c\sigma_3 + \sigma_{ci}^2} - \sigma_1. \quad (18)$$

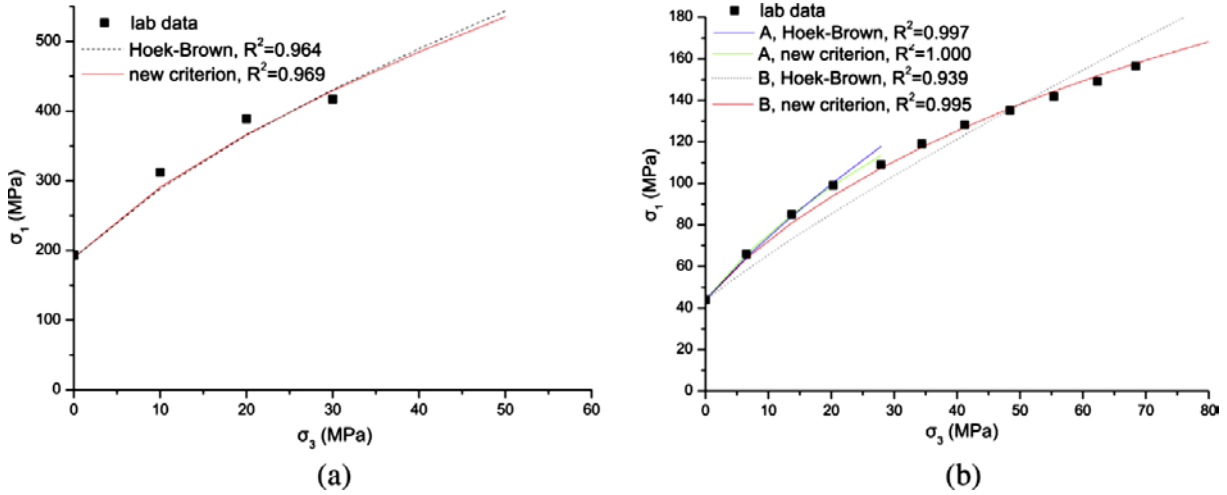


Fig. 15: Comparison between Hoek-Brown failure criterion and New Criterion for granite (a) and limestone (b) according to (Chen et al. 2016).

Fig. 15 documents the failure envelopes constructed according to equation (18) in comparison with the Hoek-Brown failure envelope (eq. (14)) for two different types of rocks tested in the lab. (Zhang 2008) extended the Hoek-Brown criterion to include the influence of the intermediate principal stress component. The developed failure criterion is called Three Dimensional (3D) Version of the Hoek–Brown strength criterion for intact rock and can be expressed as follows (Zhang & Zhu 2007):

$$0 = \frac{9}{2\sigma_c} \cdot \tau_{OCT}^2 + \frac{3}{2\sqrt{2}} m \cdot \tau_{OCT} - m \frac{\sigma_1 - \sigma_3}{2} - \sigma_{ci}, \quad (19)$$

where  $\tau_{OCT}$  is the octahedral shear stress and it is calculated as follows:

$$\tau_{OCT} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}. \quad (20)$$

### 2.1.5 Mogi failure criterion

As quite well documented by (Kwaśniewski 2013), the intermediate principal stress has some influence on the ultimate strength of rocks. Therefore, strength criteria were developed, which include the intermediate principal stress component by incorporate  $\tau_{OCT}$ . As mentioned above in the 3D extended Hoek-Brown failure criterion, (Mogi 1971) suggested a failure criterion that considered the influence of  $\sigma_2$  on rock failure.

$$0 = A \left( \frac{\sigma_1 - \sigma_3}{2} \right)^n - \tau_{OCT}, \quad (21)$$

where  $A$  and  $n$  are material parameters obtained by curve fitting. However this criterion was a subject of debate because the material parameters could not be related to the Mohr - Coulomb parameters of the Coulomb failure. Al-Ajmi & Zimmerman (2005) have deduced a linearized Mogi-Coulomb criterion as follows:

$$0 = a + b \frac{\sigma_1 + \sigma_3}{2} - \tau_{OCT}, \quad (22)$$

where parameters  $a$  and  $b$  are related to Mohr-Coulomb parameter cohesion and friction angle:

$$a = \frac{2\sqrt{2}}{3} c \cos \varphi, \quad (23)$$

$$b = \frac{2\sqrt{2}}{3} \sin \varphi. \quad (24)$$

Mogi (1967) proposed another more general failure criterion, also based on the Mohr-Coulomb criterion ( $m$ ,  $n$  and  $\beta$  are constants which have to be obtained by experimental data fitting):

$$0 = m((\sigma_1 + \beta \cdot \sigma_2 + \sigma_3) / 2)^n - (\sigma_1 - \sigma_3) / 2. \quad (25)$$

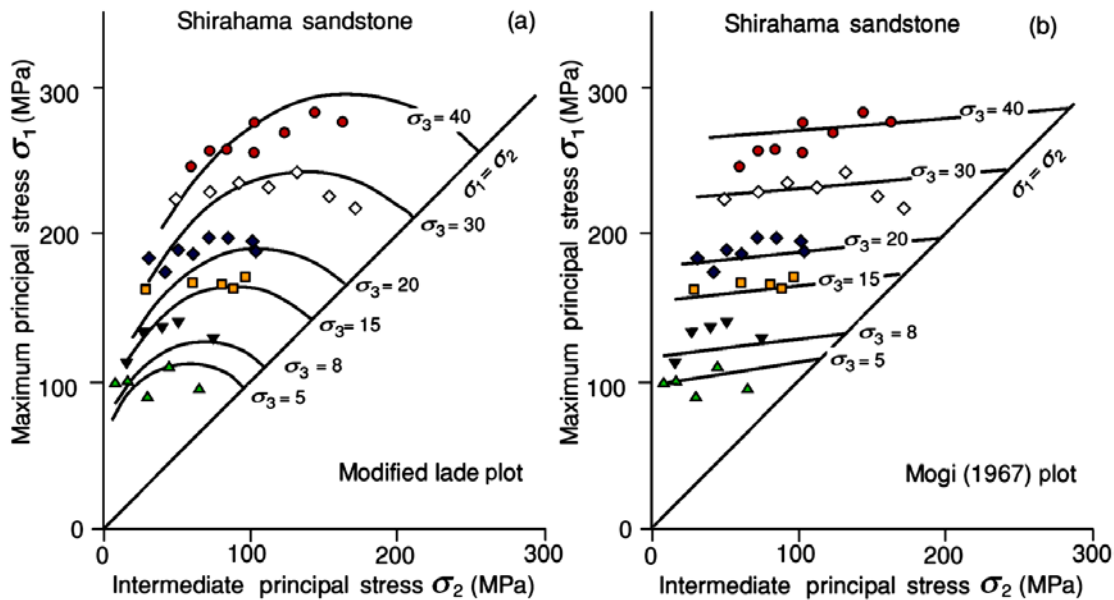


Fig. 16: Polyaxial stress data at failure for Shirahama sandstone fitted to Mogi (1967) empirical criterion and modified Lade criterion (Hackstone & Rutter, 2016).

### 2.1.6 Modified Lade criterion

Ewy (1999) presented an extended version of the Lade criterion (Lade 1977), which includes cohesion:

$$0 = \frac{\left[ \left( \sigma_1 + \frac{c}{\tan \varphi} \right) + \left( \sigma_2 + \frac{c}{\tan \varphi} \right) + \left( \sigma_3 + \frac{c}{\tan \varphi} \right) \right]^3}{\left( \sigma_1 + \frac{c}{\tan \varphi} \right) \cdot \left( \sigma_2 + \frac{c}{\tan \varphi} \right) \cdot \left( \sigma_3 + \frac{c}{\tan \varphi} \right)} - 27 - \frac{4 \tan^2 \varphi (9 - 7 \sin \varphi)}{1 - \sin \varphi}. \quad (26)$$

### 2.1.7 Modified Wiebols-Cook criterion

Zhou (1994) proposed a modified Wiebols-Cook criterion (Wiebols & Cook 1968) which is based on the Drucker-Prager criterion including the following features described by Wiebols & Cook (1968):

- compressive strength increases linearly with increasing confining stress  $\sigma_2 = \sigma_3$
- for triaxial extension stress state ( $\sigma_1 = \sigma_2$ ) strength increases linearly with  $\sigma_3$
- if  $\sigma_3$  is held constant and  $\sigma_2$  increases from  $\sigma_2 = \sigma_3$  to  $\sigma_2 = \sigma_1$ , the strength first increases to a peak value at a certain  $\sigma_2$ , and then decreases to a value greater than the starting value

The modified Wiebols-Cook criterion is given by the following equation:

$$0 = \sqrt{J_2^D} - A - B \frac{\sigma_{kk}}{3} - C \left( \frac{\sigma_{kk}}{3} \right)^2. \quad (27)$$

### 2.1.8 Griffith failure criterion

The Griffith criterion assumes that tensile failure in brittle materials initiates at crack tips. This is based on fracture mechanics and assumes tensile fracturing with uniaxial tensile strength  $\sigma_T$  as material parameter:

$$0 = 8\sigma_T(\sigma_1 + \sigma_3) - (\sigma_1 - \sigma_3)^2$$

(28)

$$\text{and } 0 = \sigma_T - \sigma_3 \text{ if } \sigma_1 + 3\sigma_3 < 0.$$

The Griffith criterion can be transformed into the shear-normal stress space:

$$0 = 4\sigma_T(\sigma + \sigma_T) - \tau^2. \quad (29)$$

## 2.2 Anisotropic stress failure criteria

Inherent anisotropy is considered as a major characteristic of rocks, in particular for metamorphic rocks due to foliation and schistosity, and sedimentary rocks due to bedding planes. From a mechanical point of view, anisotropic nature of rocks causes differences in rock strength with respect to the orientation of loading and inherent planes of weakness ( $\beta$  is the angle between plane of weakness and direction of maximum principal stress) as, exemplary, illustrated in Fig. 17 for a sample with weak plane under uniaxial compression. Fig. 18 shows tensile strength values obtained by lab tests and numerical simulations on highly anisotropic slate.

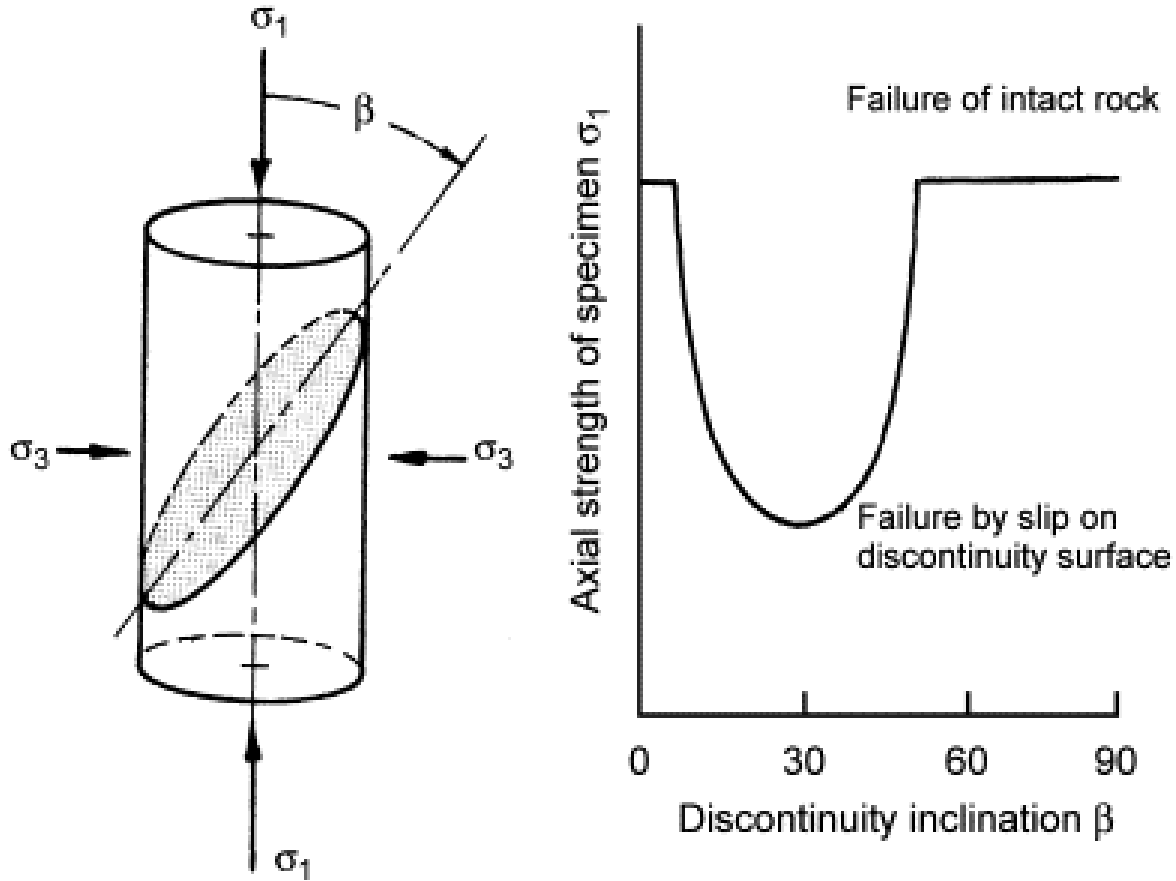


Fig. 17: Anisotropy in strength of a sample with weak plane under uniaxial compression, after (Hoek, E. & Brown 1980)

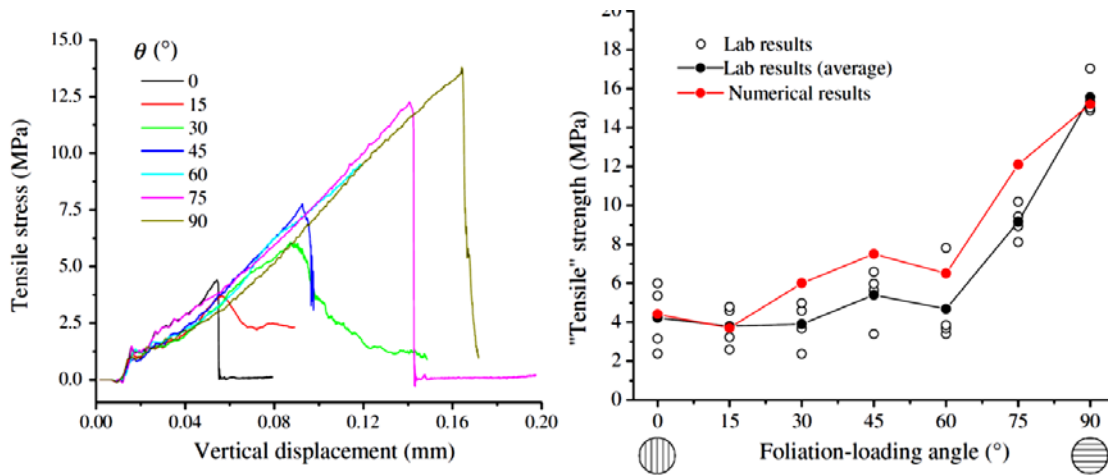


Fig. 18: "Tensile" strength of slate (Brazilian test) under different loading directions in respect to foliation (Tan et al. 2015)



### 2.2.1 Ubiquitous joint model

If only one plane of weakness exists, both, the strength of the intact rock and the strength of the weak plane have to be considered. If the principles of Mohr-Coulomb failure criteria are applied, the following two criteria have to be checked:

- Potential shear failure inside matrix:

$$0 = \sigma_1 - \frac{1 + \sin \varphi}{1 - \sin \varphi} \sigma_3 - \frac{2c \cos \varphi}{1 - \sin \varphi} \quad (30)$$

- Potential shear failure along weak plane, based on the plane of weakness theory by (Jaeger & Cook 1979):

$$0 = \sigma_3 + \frac{2(C_j + \sigma_3 \tan \varphi_j)}{(1 - \tan \varphi_j \cot \beta) \sin(2\beta)} - \sigma_1, \quad (31)$$

where,  $C_j$  and  $\varphi_j$  are cohesion and friction angle of the weak plane, respectively. Fig. 17 shows the strength as function of orientation of plane of weakness for the case of uniaxial compression ( $\sigma_3 = 0$ ) according to equations (30) and (31). In addition, tensile failure has to be checked by comparing matrix tensile strength ( $\sigma_t$ ) and joint tensile strength  $\sigma_t^j$  with minimum principal stress:

$$\sigma_3 - \sigma_t = 0 \quad \text{and} \quad \sigma_3 - \sigma_t^j = 0. \quad (32)$$

This approach can be extended to several weak planes with different orientation and strength parameters. Fig. 19 shows the uniaxial strength as function of joint inclination angle for 2 and 3 joint sets, respectively.

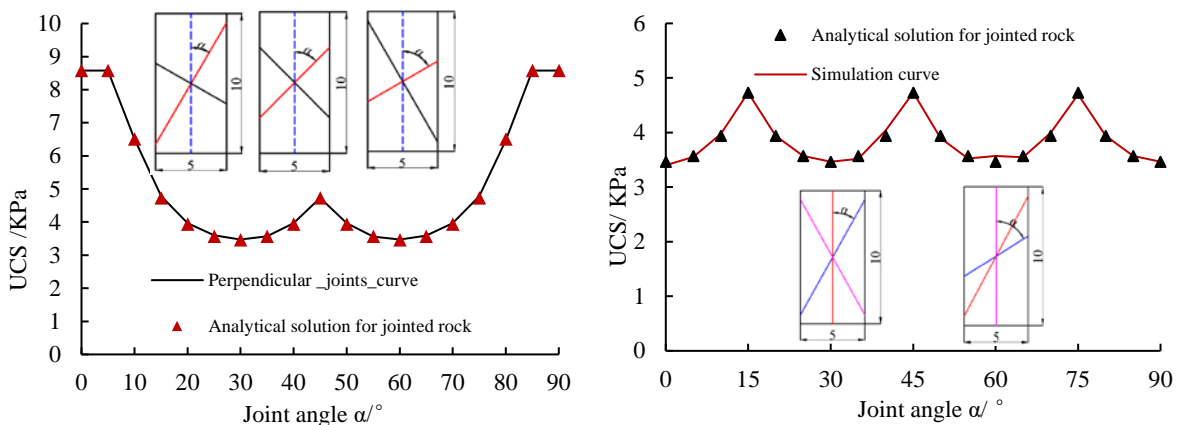


Fig. 19: Uniaxial strength vs. joint inclination angle: numerical simulation results of uniaxial compression test with two (left) and three (right) joints with identical strength parameters.

### 2.2.2 Anisotropic Hoek-Brown criteria

Colak & Unlu (2004) have introduced an angle dependent  $m_i$  value in the following form:

$$m_{i,\beta} = \left\{ 1 - A \exp \left[ - \left( \frac{\beta - B}{C + D\beta} \right)^4 \right] \right\} m_{i,90}, \quad (33)$$

where  $A$ ,  $B$ ,  $C$  and  $D$  are fitting parameters and  $m_{i,90}$  is the material parameter for rock matrix behavior ( $\sigma_1$  perpendicular to weak plane). Saroglou & Tsiambaos (2008) have proposed the following extended version (equation 34), where  $K_\beta$  depends on the angle  $\beta$  and accounts for the anisotropy:

$$0 = \sigma_3 + \sigma_{ci\beta} \left( K_\beta m_i \frac{\sigma_3}{\sigma_{ci\beta}} + 1 \right)^{0.5} - \sigma_1. \quad (34)$$

Bagheripour et al. (2011) have proposed the following expression:

$$0 = F(\beta) \left[ \sigma_3 + \sigma_{ci} \left( m_i \frac{\sigma_3}{\sigma_{ci}} + 1 \right)^{0.5} \right] - \sigma_1, \quad (35)$$

where  $F(\beta)$  is a special nonlinear function with parameters obtained by fitting lab test results. As documented by Ismael et al. (2015) all the three above mentioned approaches give similar results, whereas the criterion of Bagheripour et al. (2011) results in slightly higher strength values.

## 3 Strain failure criteria

In strain failure criteria a certain strain value is compared with a strain limit value  $FD$ . The following three expressions are simple classical strain failure criteria used in material sciences:

$$0 = \varepsilon_1 - \varepsilon_3 - FD, \quad (36)$$

$$0 = \frac{1}{3}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) - FD, \quad (37)$$

$$0 = \frac{\sqrt{2}}{2(1+\nu)} \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2} - FD. \quad (38)$$

Kwaśniewski & Takahashi (2010) have summarized strain-based criteria for rocks. They are based either on maximum principal strain (extension), maximum principal strain (compression), maximum shear strain or mean and octahedral strain. Best results for different rock types were obtained by the following expression:

$$0 = a + b\varepsilon_1 - \gamma_{OCT}, \quad (39)$$

where  $a = 0.098\%$  and  $b = 1.367$  determined by fitting of lab tests,  $\gamma_{OCT}$  is the octahedral strain and  $\varepsilon_1$  is the maximum principal strain (extensional). Failure strain for rocks is in general quite low and in most cases much below 1%. For extensional failure strain according to Stacey (1981) the following app. critical (limit) values can be given:

- Diabase, Norite  $174 \cdot 10^{-6}$
- Conglomerate  $80 \cdot 10^{-6}$
- Lava  $145 \cdot 10^{-6}$
- Quartzite  $105 \cdot 10^{-6}$
- Shale  $130 \cdot 10^{-6}$

#### 4 Energy failure criteria

In energy failure criteria a certain elastic strain energy value is compared with an energy limit value  $FE$ . The following three expressions are simple classical energy failure criteria used in material sciences:

$$0 = 0.5(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3) - FE, \quad (40)$$

$$0 = \frac{1-2u}{6E}(\sigma_1 + \sigma_2 + \sigma_3)^2 - FE, \quad (41)$$

$$0 = \frac{1+u}{6E}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] - FE. \quad (42)$$

Equations (40), (41) and (42) describe total strain energy, energy due to volume change and energy due to shape change, respectively. Xie et al. (2009) propose an energy criterion for rocks based on elastic energy and the stress deviator:

$$0 = F(\sigma_1 - \sigma_3)U - FE, \quad (43)$$

where  $U$  is the elastic strain energy given by the following formula:

$$U = \frac{1}{2E}[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2u(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)]. \quad (44)$$

Hao & Liang (2015) developed a strength criterion based on shear strain energy on the failure plane:

$$0 = \frac{(\tau + \sigma \tan \varphi)^2}{2G} \quad (45)$$

where,  $\tau$  and  $\sigma$  are shear and normal stress on the failure plane,  $G$  is the shear modulus and  $\varphi$  the friction angle.

## 5 Brittleness

The term ‘brittleness’ is mainly used to describe the post-failure characteristic (see Fig. 20). However, there are quite different definitions based on different basic parameters like:

- Strength parameters
- Shape of stress-strain curve
- Energy balances
- Elastic parameters
- Mineral composition
- Well logging data
- Friction angle
- Force-penetration graphs
- Indentation tests
- Fragmentation characteristic

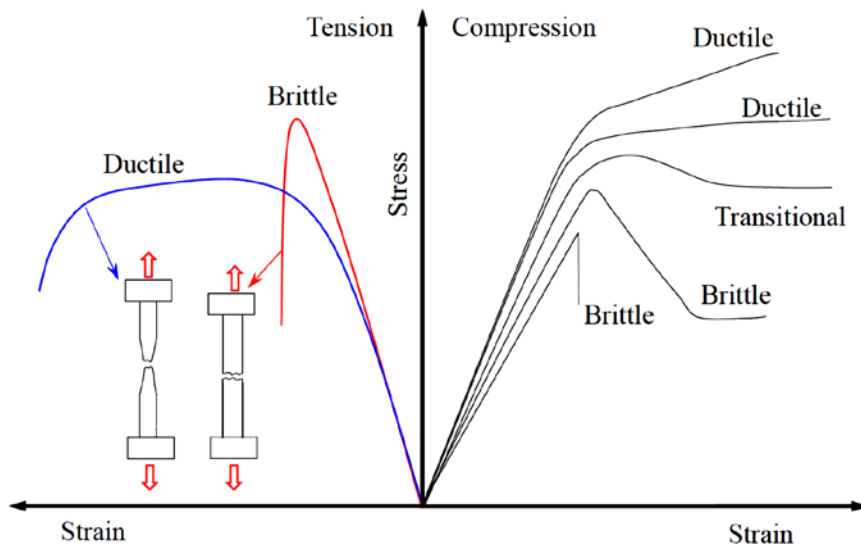


Fig. 20: Illustration of the terms ‘brittle’ and ‘ductile’ based on stress-strain-curves of compressive and tensile tests (Meng et al. 2021).

Meng et al. (2021) provide a comprehensive overview about 80 different brittleness definitions used on rock engineering. The most popular brittleness index definitions are based on a combination of uniaxial compressive strength (UCS) and uniaxial tensile strength (UTS) or Brazilian tensile strength, respectively, for instance:

$$B = \frac{UCS}{UTS} \quad \text{or} \quad B = \frac{UCS - UTS}{UCS + UTS} \quad (46)$$

More comprehensive definitions (see Eq. 47) include several parameters of the stress-strain curve like illustrated in Fig. 21.

$$B = \frac{H}{E} \quad \text{with} \quad H = \frac{\sigma_p - \sigma_e}{\varepsilon_p - \varepsilon_e} \quad (47)$$

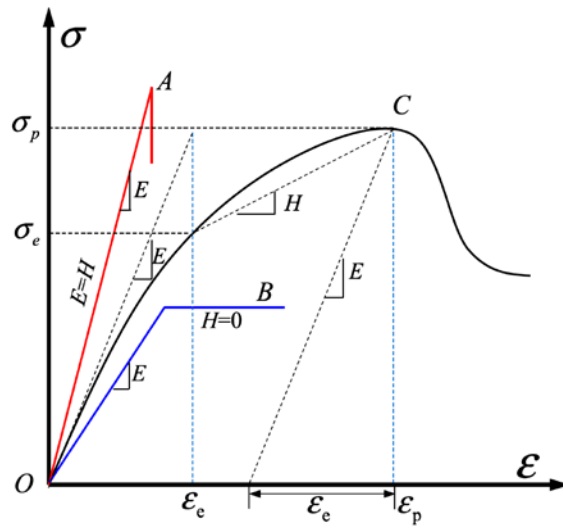


Fig. 21: Illustration of parameters used for brittleness determination according to Eq. 47 (Meng et al. 2021).

Other definitions are based on energetic considerations (see Fig. 22 and exemplary Eq. 48).

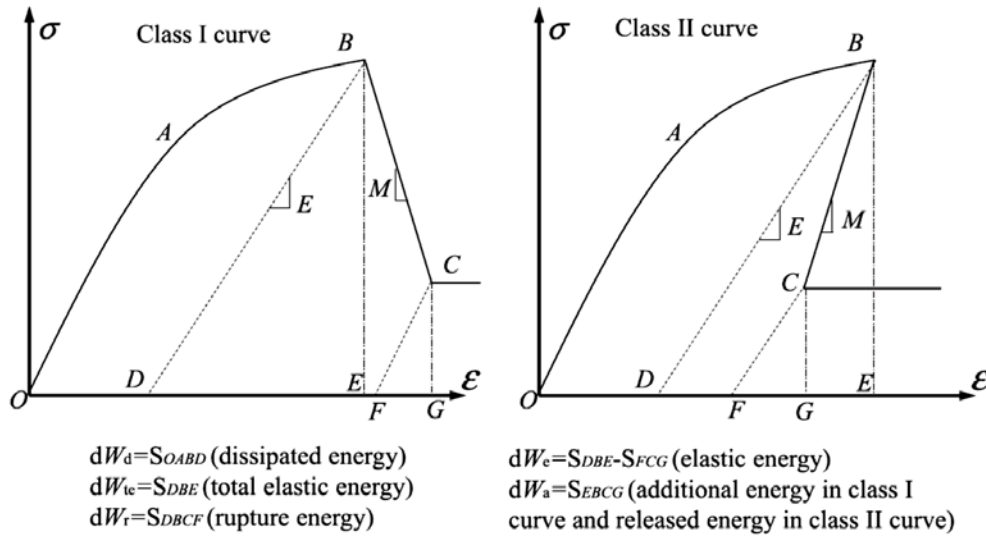


Fig. 22: Illustration of parameters used for brittleness determination according to Eq. 48 (Meng et al. 2021).

$$B = \frac{dW_{te}}{dW_d + dW_r} \quad \text{or} \quad B = \frac{dW_{te}}{dW_r} \quad (48)$$

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