UMAT Neo-Hookean Hyperelasticity

1 Theory

This routine is based on the Neo-Hookean UMAT from the Abaqus Manual [1].

The strain energy function is defined as

$$U = U(I_1, I_2, J) = C_{10} \cdot [I_1 - 3] + \frac{1}{D_1} \cdot [J - 1]^2. \tag{1}$$

The appearing invariants are those of the left Cauchy-Green tensor $\underset{\sim}{B}$:

$$I_1 = \mathrm{tr}(\underset{\sim}{B}),\ I_2 = \frac{1}{2}[I_1^2 - \mathrm{tr}(\underset{\sim}{B}^2)],\ \underset{\sim}{B} = \underset{\sim}{F} \cdot \underset{\sim}{F}^{\mathrm{T}},\ J = \det(\underset{\sim}{F}) \tag{2}$$

The cauchy stress is yield with the deviatoric left Cauchy-Green tensor $\bar{\underset{\sim}{B}} = \underset{\sim}{B}/J^{2/3}$ as

$$\sigma_{ij} = \frac{2}{J} C_{10} \left[\bar{B}_{ij} - \frac{1}{3} \delta_{ij} \bar{B}_{kk} \right] + \frac{2}{D_1} [J - 1] \delta_{ij} \tag{3}$$

The consistent stiffness is expected as

$$DDSDDE_{ijkl} = \frac{1}{J} \frac{\mathrm{d}(J\sigma_{ij})}{\mathrm{d}(\Delta d_{kl})} \tag{4}$$

The actual DDSDDE is the voigt presentation of the Stiffness C, which reads

$$\begin{aligned} C_{ijkl} =& \frac{2}{J} C_{10} \left[\frac{1}{2} [\delta_{ik} \bar{B}_{jl} + \delta_{jl} \bar{B}_{ik} + \delta_{il} \bar{B}_{jk} + \delta_{ik} \bar{B}_{il}] - \frac{2}{3} \delta_{ij} \bar{B}_{kl} - \frac{2}{3} \delta_{kl} \bar{B}_{ij} - \frac{2}{9} \delta_{ij} \delta_{kl} \bar{B}_{mm} \right] \\ &+ \frac{2}{D_1} [2J - 1] \delta_{ij} \delta_{kl} \end{aligned} \tag{5}$$

2 Usage

The UMAT can be used within Abaqus directly, uel-large-deformation and MonolithFE^2. In the latter two it needs to be compiled into with setting the MATERIAL variable to 'NeoHooke'.

This UMAT needs no internal state variables and expects exactly two material parameters: E (Young's Modulus) and ν (Poissons ratio).

References

[1] ABAQUS/Standard User's Manual, Version 6.9, Michael Smith, Dassault Systèmes Simulia Corp, 2009