

# UMAT Neo-Hookean Hyperelasticity

## 1 Theory

This routine is based on the Neo-Hookean UMAT from the Abaqus Manual [1].

The strain energy function is defined as

$$U = U(\bar{I}_1, J) = C_{10} \cdot [\bar{I}_1 - 3] + \frac{1}{D_1} \cdot [J - 1]^2. \quad (1)$$

The appearing invariants are those of the left Cauchy-Green tensor  $\underline{B}$ :

$$\bar{I}_1 = \text{tr}(\underline{B}), \quad \underline{B} = \underline{F} \cdot \underline{F}^T, \quad J = \det(\underline{F}) \quad (2)$$

The cauchy stress is yield with the deviatoric left Cauchy-Green tensor  $\bar{\underline{B}} = \underline{B}/J^{2/3}$  as

$$\sigma_{ij} = \frac{2}{J} C_{10} \left[ \bar{B}_{ij} - \frac{1}{3} \delta_{ij} \bar{B}_{kk} \right] + \frac{2}{D_1} [J - 1] \delta_{ij} \quad (3)$$

The consistent stiffness is expected as

$$DDSDDE_{ijkl} = \frac{1}{J} \frac{d(J\sigma_{ij})}{d(\Delta d_{kl})} \quad (4)$$

The actual DDSDD is the voigt presentation of the Stiffness  $C$ , which reads

$$\begin{aligned} C_{ijkl} = & \frac{2}{J} C_{10} \left[ \frac{1}{2} [\delta_{ik} \bar{B}_{jl} + \delta_{jl} \bar{B}_{ik} + \delta_{il} \bar{B}_{jk} + \delta_{jk} \bar{B}_{il}] - \frac{2}{3} \delta_{ij} \bar{B}_{kl} - \frac{2}{3} \delta_{kl} \bar{B}_{ij} - \frac{2}{9} \delta_{ij} \delta_{kl} \bar{B}_{mm} \right] \\ & + \frac{2}{D_1} [2J - 1] \delta_{ij} \delta_{kl} \end{aligned} \quad (5)$$

## 2 Usage

The UMAT can be used within Abaqus directly, uel-large-deformation and MonolithFE<sup>2</sup>. In the latter two it needs to be compiled into with setting the MATERIAL variable to 'NeoHooke'.

This UMAT needs no internal state variables and expects exactly two material parameters:  $E$  (Young's Modulus) and  $\nu$  (Poissons ratio).

## References

- [1] ABAQUS/Standard User's Manual, Version 6.9, Michael Smith, Dassault Systèmes Simulia Corp, 2009